# Investment Planning Group
## Final Report

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**OR680/SYST798 Capstone Project Course, Spring 2011**

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# 1 Executive Summary

There are many options trading strategies available to investors and fund managers. Fund managers often use multiple trading strategies because they are profitable in certain market conditions. This project analyzed a particular option strategy, the Short Strangle strategy with bull put and bear call spreads. We use this strategy to trade options on the E-Mini S&P 500, a stock market index futures contract on the Chicago Mercantile Exchange.

Fund managers often make investment decisions based on mathematical modeling, experience, and intuition. Trading models require extensive research to develop and verify useful results. As a result, they are often proprietary and confidential. Our investment planning group developed a trading simulation to find optimal trading strategies that consider optimal allocation of investment and other parameters to ensure limited risk investments.

Our work builds on the work of two previous groups[1] by extending their simulation software and models. In addition to simulation using market data, we developed a prediction model that calculates the expected profit for a particular policy using the Black-Scholes model of options pricing.

Our results show more realistic returns than the previous group's work. Optimal policies have returns of 900% over the years 2007 through 2009. We also determined the optimal values of several parameters, including the best time to trade, which is supported in existing literature.

[1] The team in fall of 2009 performed some basic analysis and produced a final report (Adamson, et al. 2009). The team in spring of 2010 wrote an initial trading simulation, which we are extending (Chen, et al. 2010).

# 2 Introduction

Each individual has a unique tolerance for risk when considering investment strategies. While a younger or wealthier individual may want to invest aggressively, an individual closer to retirement may want to sacrifice growth for security. Fund managers and individuals use these considerations when determining how to invest. The amount of risk that an investor is willing to take is central to investing and must be considered in any investment strategy.

In the fall of 2009 (Adamson, et al. 2009) and spring of 2010 (Chen, et al. 2010), project teams conducted similar risk analysis research for the short strangle strategy. The team in spring of 2010 developed a simulation tool to analyze E-Mini S&P 500 futures options trading strategies to identify potential investment opportunities. While the results were very promising, their assumptions were limited, resulting in an unrealistic trading performance. The optimal policy showed a 70,000% return over the three years of 2007 through 2009, an unreasonably large return (Chen, et al. 2010). Additionally, their tool lacked a prediction component.

## 2.1 Problem Statement

Our problem is to determine an optimal options investment strategy, balancing aggressive investment against risk of catastrophic loss, by simulating and comparing all possible policies over a period of time. Building on the work of previous teams, our tasks were to develop (1) a more realistic simulated trading process and (2) an analytical model to predict the expected profit of an investment strategy and validate the simulation.

## 2.2 Statement of Need

Investors rely on both intuition and mathematical modeling for market prediction and advising trades. However, rigorous models are often the result of extensive resources and are strictly confidential and proprietary. Operations research techniques can be used to assist decision makers to balance aggressive investment against catastrophic loss by offering scientific justification for decisions.

## 2.3 Definitions

We will define several terms that are necessary to understand the concepts in this paper. Further explanation of these terms can be found in financial engineering texts.

*Derivative*: financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables.

*Exchange*: platform where assets such as commodities or derivatives are traded in standardized contracts.

*Option*: financial derivative sold on an exchange that establishes a contract between two parties concerning the buying or selling of an asset. An option extends the buyer the right, but not the obligation to buy or sell the asset. The value of an option derives from the strike price, the spot price, the risk-free interest rate, time to expiration and the volatility of the market.

*Spot* or *Market Price*: the price of the underlying asset when an option is sold.

*Expiration Date*: date upon which the contract expires, after which it becomes worthless if it is out of money.

*Option Writer*: the trader selling the option.

*Option Holder*: the trader buying the option.

*Short Position*: in options trading refers to writing or selling an options contract.

*Long Position*: in options trading refers to holding or buying an options contract.

*Strike* or *Exercise Price*: the fixed price at which the holder can purchase (if call) or sell (if put) the underlying asset from/to the writer.

*Call Option*: affords the holder the right to buy the underlying asset from the writer at the strike price at the expiration date.

*Put Option*: affords the holder the right to sell the underlying asset to the writer at the strike price at the expiration date.

*Intrinsic Value*: the value the option has if it could be exercised immediately. A call has intrinsic value if the market price is above the strike price.

*In the Money/Out of the Money*: indicating if the option has intrinsic value or not. An option is "in the money" if it has intrinsic value.

*Stop Loss Order*: order to buy back an option once the price of the option has climbed above a specified stop price for a call (similarly for a put). This is used to minimize risk of catastrophic loss. When the price of the option has exceeded the stop price, the option is said to "stop out".

*Slippage*: the difference between intended fill prices and the price actually paid, typically due to market dynamics.

*Short Strangle Strategy*: selling both a put and a call option with the same expiration date but with different strike prices.

*Point*: basic unit for the stock market index. This has a fixed conversion to currency, but is specific to each exchange.

*Margin*: collateral to cover the risk of selling options on an exchange, usually set by the exchange.

*Terminal Wealth Relative (TWR)*: ratio of final portfolio value to initial value.

*Volatility*: measure of the uncertainty of future stock price movements.

*Implied Volatility*: volatility computed by solving the Black-Scholes equation for volatility given market premiums for an option. As compared with historical volatility which is the sample variance of past asset prices.

In this paper we will use the term *policy* for specific strangle strategy with a fixed set of parameter values. That is, when enumerating parameters for a short strangle trading strategy, each combination of parameters is a specific policy.

## 2.4 Background

### 2.4.1 Simulation Model

The previous project team, in spring of 2010, produced simulation software written in Java (Chen, et al. 2010). This program takes actual data from the E-Mini S&P 500 and simulates many trading scenarios. The results can then be analyzed using a different program, also developed by their team. Their (and our) approach was to enumerate all possible policies for a range of parameter values. The user could then analyze the expected return for a set of parameters.

Our team decided we would initially simulate the strategy using historical data and only consider developing an analytical prediction model if there was time. This turned out to be the case and we were able to develop an analytical model which determines the expected value of the profit for a particular policy. The simulation, on the other hand, enumerates all possible parameter combinations and finds the actual profit for each policy for each of the possible trading dates in the data.

The trade data provided by the sponsor includes all the option premiums for each day, expiration and strike price. Each policy chooses which of the possible options to sell, attempting to maximize the collected premium while minimizing the chance that the market will result in requiring a large payout at expiration.

### 2.4.2 Analytical Model

In addition to extending the simulation software, we implemented an analytical prediction model to find the expected value of the profit for a particular policy at the expiration date. The analytical model uses the Black-Scholes-Merton model to compute the value of an option. Then, using a motion model for the underlying asset price, we determined the probability distribution for the price at the expiration day. By integrating the profit of the policy over the probability distribution we found the expected value of the profit of that policy. This enables a trader to predict the value of a policy given some idea of the market in the future without having to simulate or rely on past performance.

### 2.4.3 E-Mini S&P 500

The E-Mini S&P 500, or E-mini, is a stock market index futures contract traded on the Chicago Mercantile Exchange. It is comprised of a subset of companies on the S&P 500 index. The main benefit of the S&P is that it is one of the most liquid and rational markets (Coval and Shumway 2000). It is also thought by some to be the most profitable for writing options (Coval and Shumway 2000). An additional benefit of using the E-mini is that it is traded electronically and therefore accurate pricing data is available. We did not have access to the tick-by-tick data and as a result only traded daily using the end-of-day pricing data.

The options traded on the E-mini expire monthly on the third Friday. The options are contracts on the quarterly futures contracts. Since the E-mini is an index, there is no underlying commodity exchanged on expiration. If the option expires in the money, the option writer buys it back at a price equal to the difference between the strike price and the current index price. Each point on the E-mini index is worth $50.

In this way, options are typically used for hedging against risk of a loss on a more basic asset. The writer charges a premium to sell the option. If the index price goes above the strike price on a call, the buyer makes a profit proportional to the index price, and similarly for a put if the index price goes below, the buyer makes profit. If the option expires out of the money, the writer keeps the premium.

#### 2.4.4 Short Strangle Strategy

There are many different trading strategies, with different approaches to mitigate risk. Two strategies for trading options are the strangle and straddle strategies. Both of these involve selling a pair of put and call options. A straddle strategy consists of selling (or buying) the two options with the same expiration date at the same strike price. However, all of our work revolved around analyzing the strangle strategy – trading both options with the same expiration date but at different strike prices. Typically these options are initially traded out of the money; that is, they have no intrinsic value. The short strangle has been shown to be more profitable than other strategies in the E-mini S&P 500 (Coval and Shumway 2000).

The short strangle has limited profit and unbounded loss. It incorporates the simultaneous selling of a put option and a call option, each out of the money with different strike prices. Since both the put and call are out of the money, the strike price for the call is above the spot price and the strike price for the put is below the spot price. The short strangle is most profitable when the expiration price falls between both strike prices.

The following figure shows the profit to the writer as the price of the asset increases. $P_1$ is the strike price of the put and $C_1$ is the strike price of the call. As the index price at expiration (the horizontal axis) increases to the right, the profit (the vertical axis) to the writer varies. If the expiration price is outside of the strike prices of both put and call, profit decreases until the cost of buying the options back is greater than the premium collected, resulting in a net loss to the writer. Note that profit can be negative and the loss is unbounded.

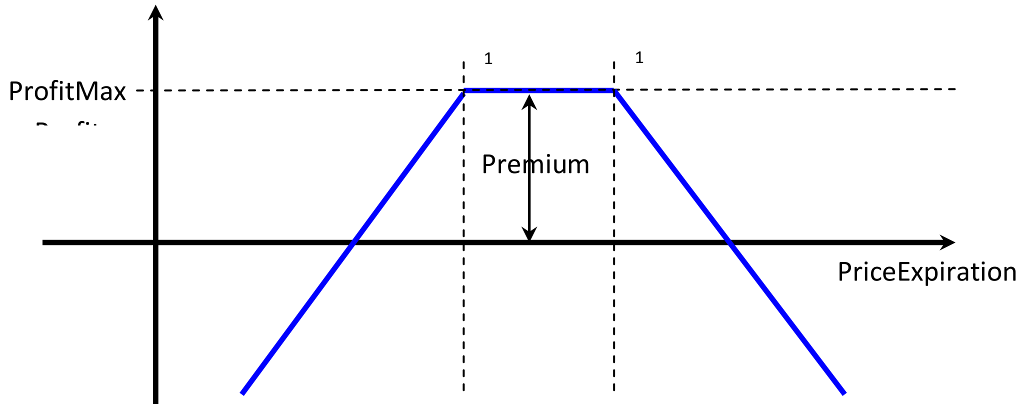


**Figure 1 Profit spread for a short strangle strategy**

The long strangle strategy is similar to the short strangle except that it takes the opposite position. The profit is then flipped over the horizontal axis to show that the buyer profits most when the seller loses most.

### 2.4.5 Iron Condor (Bear-Call/Bull-Put)

The previous team used stop-loss orders as a loss mitigation method. One of the possible parameters in their strategy was the number of points away from the traded put or call prices before the stop order took effect. A higher stop-loss parameter resulted in greater potential loss. A lower stop-loss parameter may limit the potential profit. A stop-loss order complicates both the analysis and simulation. Stop orders are ordinary market orders and therefore may experience slippage, which must be modeled in the simulation. An option writer might also want to sell an additional option immediately at a higher strike price if one of the options strikes out.

Our strategy simplifies this approach by using a spread option strategy called the bear-call/bull-put or iron condor. The iron condor is the simultaneous entering into four options contracts that include a short strangle strategy and a long strangle strategy. The long strangle strategy is further out of the money than the short strangle. The objective of the bear-call and bull-put is to limit the loss.

A stop-loss order takes effect when the option price climbs above a stop-loss threshold at any time before expiration. Depending on the volatility of the market, time to expiration and the stop-loss spread, this may be more or less likely. The bear-call/bull-put spread option strategy is a passive form of insurance for the writer. By buying a separate pair of options at the original stop-loss thresholds, the trader can guarantee that at expiration, their loss will be limited. Not only is the loss limited, but they will not have to pay the full loss even if the index price rises (or falls) beyond the bear-call or bull-put at any point before expiration. Only the price at expiration matters. A speculator's maximum profit would be the premium that they collect minus the price of the long strangle options. The premiums on the long strangle contracts are lower than the short strangle prices since they are further out of the money.

The profit function of the iron condor strategy is the combination of the four profit spreads associated with the four contracts. These four contracts are:

1. Sell one out of the money call option
2. Buy one further out of the money call option
3. Sell one out of the money put option
4. Buy one further out of the money put option

Suppose $C_1$ and $P_1$ are the strike prices of the short call and put options, respectively. Then $C_2$ and $P_2$ are the strike prices of the long call and put, respectively; $C_2 > C_1$ and $P_2 < P_1$. In the following figure, $S_T$ is the price of the index at expiration. While the index has crossed above the bear-call $C_2$, the option does not experience a loss, as a stop order would. At expiration, all options are out of the money and worthless. Therefore, the profit is the premium of the short put and call options minus the premiums of the long put and call options.

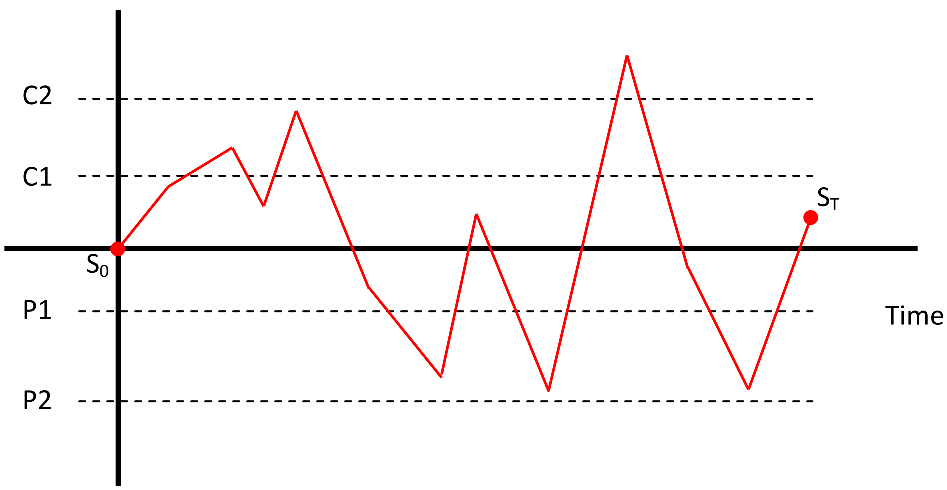


Figure 2 Example of the Iron Condor strategy as the index price varies over time

The following are the profit functions for each option in the Iron Condor strategy. Let *p* be the premium for each option.

1. Short call: $p - \max\{ S_T - C_1, 0 \}$
2. Long call: $\max\{ S_T - C_2, 0 \} - p$
3. Short put: $p - \max\{ P_1 - S_T, 0 \}$
4. Long put: $\max\{ P_2 - S_T, 0 \} - p$

The total profit is simply the sum of these four functions. The figure below shows the profit as a function of the index price $S_T$. The chart is similar to the one for the short strangle above. While the loss for the short strangle is unbounded as the index price goes up or down, both profit *and* loss are bounded for the bear-call/bull-put strategy. The profit is bounded above by the premium collected for the iron condor and the loss is bounded below by the strike prices of the bear-call and bull-put.

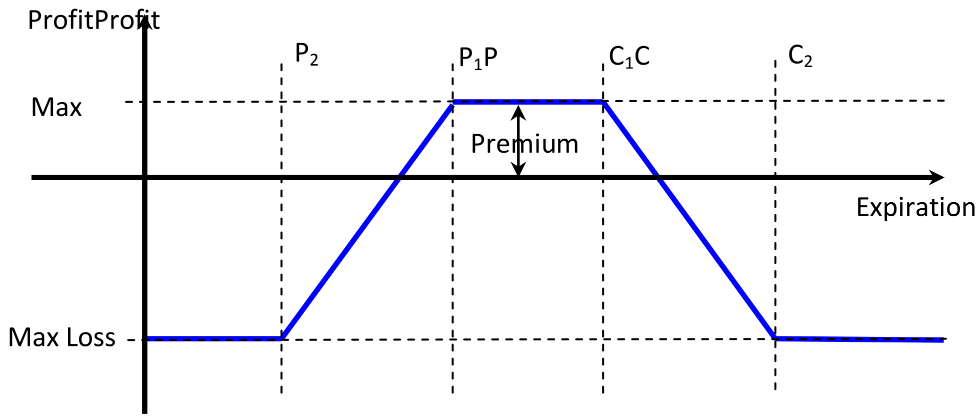


Figure 3 Profit spread for the Iron Condor strategy

### 2.4.6 Strategy Parameters

A strategy can be parameterized in many different ways. The trader must first decide when he will trade. Since we take the position of the seller, we will decide how early to sell the options. This is measured in calendar days before expiration.

The second set of parameters is the strike prices of the options. The put and call options are not necessarily equally spaced around the spot price. The previous team simply had two parameters, one controlling the strike price of the call and another for the strike of the put. Both ranged in [5, 25], meaning the call would have a strike price 5 to 25 points above the spot price, and a put would have a strike price 5 to 25 points below. Both parameters have the same range, but they are independent and need not have the same value.

| Strategy Implementation | Days | Put Strike | Call Strike | Stop-Loss | Premium | Bear-Call Increment | Bull-Put Increment |
|---|---|---|---|---|---|---|---|
| 2010 | 42 | -15 | 5 | 20 | *not used* | | |
| 2011 | 28 | *not used* | | | 20 | none | 55 |

Table 1 Parameters for each of 2010 and 2011 strangle strategy implementation

Our new strategy does not parameterize over strike price, but rather the premium. For each option and strike price, the market determines the premium. We parameterize over the premium and for each value of premium, find the option (with strike price) with a premium closest to this value. Both put and call options are sold for this premium, meaning the strike prices may be close to symmetric, but are not necessarily so.

We also consider the maximum volatility for any trading day. If the market volatility is higher than some threshold, no trade will be executed. In most of our analyses, this parameter is not used and trades are executed regardless of the market volatility.

An additional parameter the previous team used was the stop-loss condition. Since this is replaced by the bear-call and bull-put, we do not use this as a parameter. Instead we have a similar parameter which determines how close the bear-call strike price is to the strike of the call. Similarly, there is a separate parameter that determines how close the bull-put strike price is to the put. These two parameters range over the same values, but are independent. For some strategy, it may be that the bear-call spread is very large while the bull-put spread is much smaller. This strategy would perform best in a bear market.

### 2.4.7 Black-Scholes-Merton Model

The Black-Scholes-Merton model (or Black-Scholes) is a popular model for pricing options. At its core, the Black-Scholes model is a linear partial differential equation describing the relationship between the price of an option, the underlying asset and the risk-free rate of return (Hull 2012). The Black-Scholes formula is below

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

where $S$ is the current price of the underlying asset, $V$ is the price of the option, $r$ is the risk-free rate of return and $\sigma$ is the volatility. This differential equation is then solved for a European put or call option. The derivation is out of the scope of this paper, but we will show the pricing formula for European put and call options.

Let $S$, $r$ be defined as before. $K$ is the strike price and $\tau$ is the time until expiration. Then

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$C = S\Phi(d_1) - Ke^{-r\tau}\Phi\left(d_1 - \sigma\sqrt{\tau}\right)$$

$$P = -S\Phi(-d_1) + Ke^{-r\tau}\Phi\left(-d_1 + \sigma\sqrt{\tau}\right)$$

where $\Phi$ is the cumulative standardized normal distribution function. Using these two formulae, one can compute the price of option in a rational market. Of course, the actual market value of an option is usually slightly off. There is always a bid-ask spread and other market conditions, such as volatility smiles, can also affect the price of an option (Hull 2012). There are several basic assumptions that need to be satisfied in order for the Black-Scholes model to be effective (Wilmott 1999).

- The underlying asset follows a lognormal random walk
- The risk-free interest rate is a known function of time
- There are no dividends on the underlying asset
- Delta-hedging is done continuously
- There are no transaction costs on the underlying
- There are no arbitrage opportunities

Not all of these are valid for our analysis – particularly the delta-hedging and transaction costs. We can reasonably assume the index follows a lognormal random walk. We assume the interest rate is constant for the times we are considering. There are no dividends on the E-mini. And it's reasonable to assume no arbitrage opportunities.

### 2.4.8 Geometric Brownian Motion

The market is a random process and cannot be predicted. But its process can be described. One of the easiest ways to describe the market is to use Geometric Brownian Motion. While Brownian motion assumes the next value is normally distributed around the mean, Geometric Brownian motion assumes the percent change in the next time-step is normally distributed (Wilmott 1999). A random variable that follows this process is called a log-normal random variable. Specifically, if $S$ is a log-normal random variable then

$$\frac{\Delta S}{S} = \varphi(\mu \Delta t, \sigma^2 \Delta t)$$

where φ is the normal probability density function, μ is the expected return on the stock and σ is the volatility of the stock price (Hull 2012). Given a stock price $S_0$ at the current time and some future time $T$, the distribution of $S_T$ is the following (Hull 2012):

$$\ln(S_T) \sim \varphi(\ln(S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T)$$

This gives us a model for the prices of the index that we can use to find expected profit of a policy.

### 2.4.9 Scope

Hedge funds and large traders spend vast resources developing and tuning financial models. In order to make progress, we limited our scope significantly. In many ways these limitations exist only to focus our analysis, but in some cases we do make simplifying assumptions.

We performed all of our analysis using European put and call options, which cannot be exercised except on the date of expiration. American options, on the other hand, can be exercised at any time. European options are much easier to analyze analytically and somewhat simpler to simulate. Unfortunately, we only have data for American options (Chen, et al. 2010). Because American options can be exercised at any time, they are of more value to the holder and the premium is higher. So we will simulate and analyze European options using the slightly higher premium of American options.

We traded only once for each expiration date without any hedging. Once the trade is executed there is no action on the trade until the day of expiration. This is called trade-and-forget.

None of our results, including final TWR, took the risk-free interest rate into account. Ideally, the final return would be adjusted down by the interest rate to show marginal return on investment.

Our simulation and analyses take the view of the options writer. We optimize for the profit of the seller, where the writer collects premium as income and possibly experiences loss when buying back the option on expiration.

Finally, we only considered closing prices. This doesn't affect the results very much since we only trade once per expiration date. Premiums for options do not vary significantly day-to-day until expiration is very close. If, on expiration day, the index price has a very large spread, when the holder decides to exercise the option has a significant effect on the final cost to the writer.

# 3 Requirements and Tasks

Throughout the course of this project, our team met with our sponsor to review status updates as well as to learn and discuss necessary financial concepts. Our project requirements and tasking were developed over the first several weeks during these sponsor meetings. Below, we describe the set of tasks that we accomplished in mostly chronological order.

## 3.1 Simulation Front End

Our first task was to improve the simulation software and add a graphical user interface (GUI) front-end. The existing analysis software had a GUI for the user to interact with the data and analyze it. However, this analysis program simply loaded results from a file that was previously generated. To run the simulation and generate results, the sponsor needed to modify the source code, compile and then rerun the application. Our team started by creating a new user interface so that any user can modify the parameters and rerun the simulation, generating new results. We then integrated the GUI front-end with the analysis software and the result was a single stand-alone application that could both run the simulation and analyze the results.

For more information about how the user interacts with the software see Appendix 11. Our team did not implement all of the functionality, some of it was carried over from previous work.

### 3.1.1 Runtime

In our first meeting with our sponsor, he mentioned that the simulation took a very long time to process – sometimes up to three hours on his computer. Once we created a simulation front-end, we took a look at the running time of the simulation itself. By modifying the control flow we were able to get a speedup of 14x for a single CPU. Parallelizing the execution of the simulation using multiple CPUs dropped the running time by a factor roughly inversely proportional to the number of CPUs. This significantly increased our effectiveness at simulating different strategies and analyzing the results quickly. Using our own multi-core hardware, we cut simulation runtime for the initial default trading policy from approximately 45 minutes to less than 1 minute.

## 3.2 New Strategy Implementation

The software as we received it only simulated the previous team's strangle strategy. Our team's new strategy used the previous code and extended it. The basic requirements were to use premium as a parameter, use bear-call/bull-put instead of a stop-loss, include Kelly's Criterion when computing the optimal fractional allocation of investment, use a slippage model to limit trade size and incorporate new data. We will define each of these next.

### 3.2.1 Premium as a Parameter

The previous strategy parameterized over the strike prices of the put and call. Each put and call strike price was given a range. Technically, the difference between the strike price and the spot price was within a range. Our new strategy would not specify the strike price difference as the range, but rather the premium.

The reason we use premium as a parameter is that a premium estimates risk. A put and call with the same premium have roughly the same risk as valued by the market. If both the put and call have the same premium, the risk of the written strangle is balanced and we called it premium neutral.

The new premium parameter has a default range of [5, 25] points with a step size of 1 point. For each premium value, the simulation looks up the strike price in a table. Since the market determines the price of each option, based on days to expiration, volatility and other market conditions, this could not accurately be calculated. Once the corresponding strike price is determined for the given premium, that option is then used in the trade.

Our premium parameter is one-sided, in that the put and call each have the same premium, which is the value of the parameter. The parameter is not the sum of the two premiums.

### 3.2.2 Bear-Call/Bull-Put

As explained above, the previous strategy implementation used a stop-loss parameter. One of our requirements was to replace the stop-loss parameter with a new strategy, the bear-call/bull-put. This is explained in section 2.4.5.

The parameter for a bear-call/bull-put strategy is the number of points difference between the strike price of the short call and long call, and similarly between the short put and the long put. These two differences are independent. The bear call may be 50 points above the call while the bull put is 10 points below the put. In our simulation, both of these differences have a default range of [5, 100] with a step size of 5.

### 3.2.3 Kelly's Criterion

Traders do not wish to expose themselves to too much unnecessary risk. They typically do not invest 100% of their capital in a single trade. One step in the analysis of these policies is to determine which is the optimal fraction of capital to invest. If a trader invests 50% of their capital on each trade, the gains will be less than if they had invested 100%, but if they lose a single trade, the losses are not so devastating.

The previous team implemented a simple method to find the optimal fraction of allocation (Chen, et al. 2010). For each strategy and its performance on the historical data, they determined the best possible allocation fraction, 5% to 100% in increments of 5%. The goodness measure is the TWR. The fraction with the highest TWR is picked as the best fraction. Each strategy has only one best fraction of allocation in the analysis.

It should be easy to see that strategies that perform very well will have an optimal allocation fraction of 100%; and poor-performing strategies will recommend investing only 5%. But investors cannot tell whether a policy will perform well or poorly, they only have the historical data. The Kelly Criterion is a way to estimate the optimal fraction using the results of the trading history.

The Kelly Criterion was a formula developed by John Kelly Jr. to determine optimal size of a series of bets (Nosek 2005). It is now widely used in investing strategies. The formula determines the amount of capital to invest in any trade given the amount of risk the trader is willing to assume.

Kelly's formula is equal to the ratio of expected value of the amount invested to market expectation of how much will be earned if an investor profits (Wikipedia 2011) (Kuepper 2004).

$$K = \frac{b \cdot p - (1 - p)}{b}$$

where *p* is the probability that a trade is profitable and *b* is odds on the investment, or the payoff if won. Both *p* and *b* are calculated from historical trades. *K* is the percentage of capital that should be invested in the trade. Note that *K* can be negative, meaning the investor should take the opposite bet. However, in our case that simply means the investor will not trade.

As we calculate the final TWR, the fraction of capital for each trade is determined from the past performance of that policy itself. Initially, the policy has no history and we use a fixed prior. As more history becomes available, the prior becomes less and less important.

We implemented this by assuming some prior trade history to bootstrap the computation of Kelly's Criterion. Analyzing the previous year's results on a set of strategies, we determined that the expected probability of wins was 60% with an average winning payoff of 23 points with an average loss of 21 points. This was incorporated into the analysis software and included in the computation with each 5%, 10%... fractional allocation. Of course, as the analysis computes the returns for each trade, it includes the prior and the history for that particular strategy. Eventually, the history strategy outweighs the prior and poor strategies will tend to recommend extremely low investments while good strategies approach 100%. However, good strategies will never reach 100% fractional allocation without a perfect trade history.

### 3.2.4 Slippage Model

Slippage is the difference between the expected filled price of a trade and the actual filled price. Slippage is due to several factors, such as the time from when an order is requested until it is executed, the volatility in the market and the size of a trade relative to the market. If the market is quite small, and a trader attempts to sell a large number of options, the price of each option will end up trading significantly lower than it did at the start of the trade. This indicates that even if the investor was willing to invest 100%, investing at only 80% would provide a greater payoff because the transaction cost due to slippage is significantly lower.

Slippage will reduce the profit of any trade, and significantly reduce profit for larger trades. The previous team assumed a fixed slippage, but generally had this set to 0. We implemented a model to incorporate slippage into both the selling and buying of the options, using the actual volume data from the E-mini.

The slippage model we implemented in software is described in detail in Appendix 8. There are essentially two parts to slippage, permanent slippage due to market volatility and immediate slippage due to the reaction of the market to the trade size. The investment fraction, initial investment amount and margin determine the number of options to sell, and we had options data which provided us with the total number of options sold on each particular day. This gave the total market size. Using these two numbers, we calculate the slippage due to the size of the trade.

#### 3.2.5 New Data

The software requires several datasets for use in the simulation. The primary dataset contains the options prices for each option from 1997 to 2009. However, many of these early years are very sparse (Chen, et al. 2010). Other files being used are the volatility of the S&P 500 and the futures prices for the E-Mini S&P 500. The volatility for the S&P 500 applies equally well to the E-mini, so this was not an issue. However, no data was originally available for the price of futures on the E-mini. The previous team used the actual price of the S&P 500 index (Chen, et al. 2010).

We were able to find and purchase the futures data for the E-Mini S&P 500 and incorporate it into the simulation (Normal's Historical Data 2011).

Also for the prediction model, we used the LIBOR interest rates for use in the Black-Scholes model. This data was freely available online (LIBOR Rates History 2011).

#### 3.2.6 Simulation Approach

The simulation software implementation was straightforward. The previous team wrote a Java program to enumerate all combinations of parameter values and created a new policy using those values. The resulting policy uses trade data for some time range, usually 2007 through 2009 since that was the time range with the highest-quality options data. The policy, for each monthly trade it executed, determined the payoff from trading a single strangle.

The only significant change to the strategy process is that instead of checking the price of the index at each day before expiration for a potential stop-order, we simply bought the bear-call and bull-put options. This eliminated the need to check each day’s trading price. Instead we determined the premium on the day of the initial trade and then using the expiration price, computed the payoff for each option.

In our case, the payoff is the premium from selling the short strangle pair plus the value of the bear-call and bull-put options on expiration minus both the premium from buying the bear-call and bull-put options and the cost to buy back the short strangle pair at expiration. See 2.4.5 for the payoff functions.

Another change to the simulation is the ability to determine the strike price on the short strangle by using a premium parameter. This was a simple lookup in the table of options data from the Chicago Mercantile Exchange (CME). Since the bear-call and bull-put options may be very far away from the spot price, there may be no data to find the premium. If there is no data, we use the Black-Scholes model to compute an estimate of the premium.

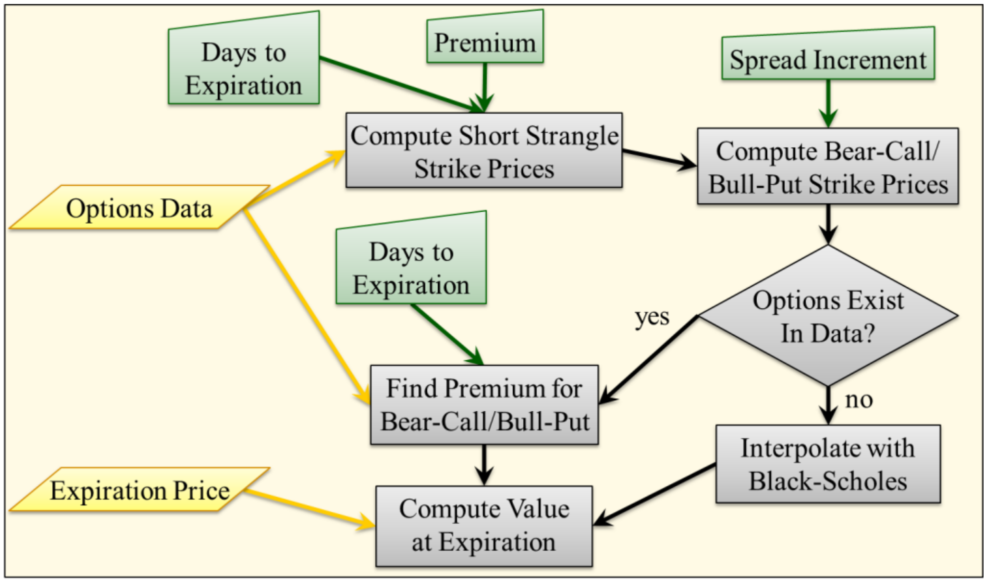


Figure 4 Simplified flowchart for the simulation showing how parameters and data contribute to the control flow

In the analysis portion of the software, the simulation results are used to compute the results of trading using some fixed initial investment amount. This code also computes some basic results such as TWR, risk of ruin and optimal investment allocation. The simulation itself only computes the payoff for a single pair of options. This code is valid for the previous team's strategy, but not once slippage is implemented. With slippage, the payoff can change significantly depending on the size of the trade relative to the market.

Our team modified the analysis code to incorporate slippage into the calculation of TWR. In turn, this affected the risk of ruin values and optimal allocation of capital to investment. The method to determine the optimal fraction allocation is also exhaustive enumeration. The software lists all possible investment fractions, from 5% to 100% in increments of 5% (and now also Kelly's fraction) and attempts to trade with each of those percentages. The outcome and payoff of each trade is already computed in the simulation, the analysis needs only to apply it to see the rate of growth.

## 3.3 Prediction Model Implementation

### 3.3.1 Finding Strike Price Using Black-Scholes-Merton

The requirement for the prediction model was to find the expected profit for a strategy, assuming the Black-Scholes-Merton model. As described in 2.4.7 and 2.4.8, the Black-Scholes-Merton model assumes the underlying asset price at some future time $T$ follows a log-normal distribution. To compute the expected value of the profit of a strategy we can integrate the profit over the probability distribution of the asset price.

Similar to the strategy simulation, the prediction model begins by enumerating all possible parameters and generates a policy with each combination of parameter values. For each policy, the software then iterates over all trades within the time period, typically 2007-2009. This three-year period has 36 expiration dates and so 36 opportunities to trade.

First the prediction model needs to find the options to trade. Using the spot price and a parameterized premium, the software uses the Black-Scholes model to solve for the options strike price. There is no easy way to invert the put or call option pricing formulae in 2.4.7. Instead we fix the volatility, interest rate, time to expiration and premium and solve for strike price numerically using a root-finding method. In our case, we used Newton's method. Newton's Method is an iterative technique that constructs a sequence on the strike price $K_n$ that in general converges quadratically towards $K$. The sequence is defined as:

$$K_{n+1} = K_n - \frac{f(K_n)}{\dot{f}(K_n)}$$

where $f(K)$ is the pricing function for a European put or call option minus the premium and set equal to zero and $\dot{f}(K)$ is the derivative of $f(K)$.

There are four options to trade in the iron condor strategy, but only the short put and short call are a function of premium. Once we have the short strangle strike prices, we can calculate the long strangle strike prices of the bear-call and bull-put. However, when using the premium to find a strike price, the value for the strike is in general not a multiple of 5. Since all options sold on the market have an increment of 5 points, we find the closest valid strike to the calculated strike price found above and then re-compute the premium for the short strangle strike prices using Black-Scholes-Merton. This is similar to the approach used by the trading simulation in 3.2.6. Then the bear-call and bull-put strike prices are fixed and their premiums are also computed using Black-Scholes-Merton.

### 3.3.2 Computing Expected Profit

The next step in the process is to find the payoff function $h(S_T)$ using the strike prices of the four options. The payoff function is the sum of the four functions described in 2.4.5. As the expiration price varies, some of the options become more valuable and others become less valuable. In the following figure, an example index price distribution is shown in grey in the background. Since it is centered slightly upward, higher than the spot price, assuming the short strangle was symmetric around the initial spot price, this is a bull market. The short call will likely be in the money, while the put will be out of the money. In this case, the expected value of the strategy will be slightly less profitable than if the mean of the stock price were centered between the short strangle.

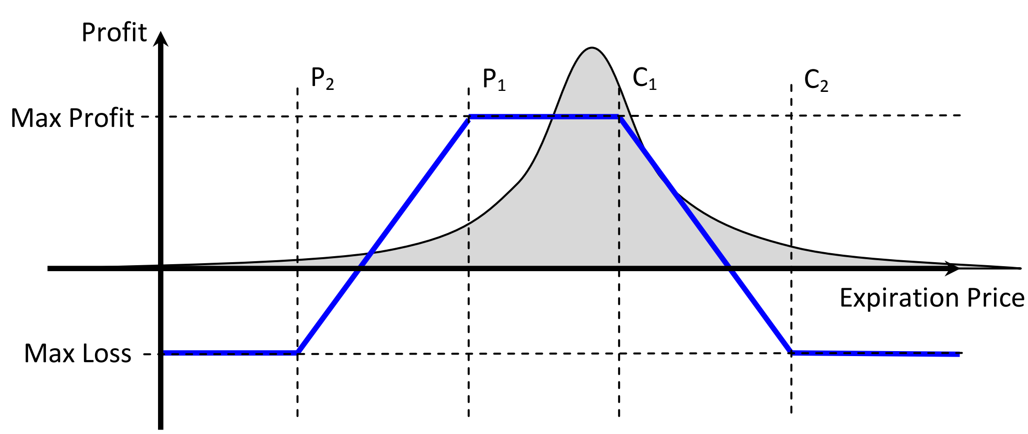


**Figure 5 Iron Condor profit spread with example distribution on expiration price**

To compute the expected value of the profit of a policy we use the lognormal property of the distribution of asset price at some future time *T* as described in section 2.4.8. Defining a new random variable *Y*:

$$Y = \ln(S_T) \sim N\left[\mu_y, \sigma_y^2\right]$$

$$\mu_y = \ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T$$

$$\sigma_y^2 = \sigma^2 T$$

The expected value of the profit of a policy is the inner product of *Y* and the payoff function $h(S_T)$ where $S_T = e^Y$:

$$E[h(S_T)] = \int_{-\infty}^{\infty} \varphi\left(y, \mu_y, \sigma_y\right) \cdot h(e^y) dy$$

Since integrating from negative infinity to positive infinity is not technically feasible and also *Y* is normally distributed, we can instead approximate the expected value with arbitrary accuracy by integrating symmetrically about the mean of *Y* by $n\sigma_y$:

$$E[h(S_T)] \approx \int_{\mu_y - \mathrm{n}\sigma_y}^{\mu_y + n\sigma_y} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-\left(y-\mu_y\right)^2}{2\sigma_y^2}\right) \cdot h(e^y) \cdot dy$$

The expected value calculation in the prediction model software uses an implementation of Simpson's rule to integrate the profit function numerically. Since the prediction model is currently iterating over real data, the software also calculates realized profit as if the policy were actually traded at those prices. And finally, the prediction model compares both the expected value of the profit and the realized profit against the profit from the simulation results for that policy and trade. The profits from the simulation results are usually just slightly different than the realized profit using the prediction model. This is due to the fact there are slight differences between both the options strike price and premium when computed using Black-Scholes and when determined by the options data which reflects actual market conditions.

# 4 Results

We present here the results of both of our models – the strategy simulation using actual market data and the prediction model using an analytical approach. The results for the simulation show how a particular policy performed on an actual time-series of market prices. Aggregating the results over three years increases the confidence in the overall performance, but does not guarantee that the performance will be comparable in similar market conditions. The market is simply too unpredictable for that.

On the other hand, the prediction model assumes a motion model for the market and uses it to compute an expected return for the strategy. Since it also enumerates all parameter values and iterates over the strategies, it can be compared loosely to the results of the simulation. Roughly, one can view the simulation results as how the strategy performed compared to the expected performance in any market.

Because both the simulation and prediction model exhaustively enumerate the available parameters, we could not allow full freedom for all parameters as the resulting datasets would be too large. We decided to use the years of 2007, 2008 and 2009 as the baseline years for analysis. And in general we did not limit trading based on volatility.

## 4.1 Strategy Simulation Results

Our strategy is analyzed based on the Terminal Wealth Relative (TWR) criterion. This is simply the ratio of the final value of the portfolio to the initial value. In effect, it is the multiplier on your investment. However, this is a raw measure of return that does not take into account the volatility of market. The Sharpe Ratio is a better measure of the return accounting for volatility of returns. A policy that performs consistently well is better than a strategy that is sometimes great and sometimes poor. The Sharpe ratio is

$$S = \frac{E[R - R_f]}{\sigma}$$

where $R_f$ is the risk-free interest rate, $E[R\text{-}R_f]$ is the expected marginal return and σ is the standard deviation of the returns (Wikipedia 2011). For policies that perform consistently well, the Sharpe Ratio will be high. Note that since we do not have the risk-free rate of return, we use the previous team's calculation of the Sharpe Ratio as the average profit over the standard deviation of profit. We call this the Modified Sharpe Ratio.

### 4.1.1 Overall Optimal Strategy Parameters

Our team wanted to compare our strategy against the strategy of the previous team. Since our intention was to make the simulation more realistic, we expect the TWR of our optimal strategy to be significantly lower than theirs. The previous team reported an optimal policy with a final TWR of approximately 711 (Chen, et al. 2010). Unfortunately, we could not reproduce their results using the code we were given. When we ran the software as initially provided, the best policy showed a TWR of 699.61, very close to the reported optimal 711. This was using Dataset F, except slippage was not implemented yet. We used this result as our baseline and the TWR against which we would compare the new assumptions.

Of course, a return of 70,000% over three years is unrealistic for two reasons. First, the simulation chooses the best possible policy knowing the future market prices. While this is unrealistic, we have this assumption in all of our analysis; it's simply the way the simulation runs, trying all possible parameters on the data and picking the best combination. The second reason this isn't realistic is that this doesn't incorporate slippage. The policy invests 100% each trade. Starting with an initial investment of $1 million and ending up with $700 million means that on any single trade, the $700 million investment will most certainly move the market and cause the trade to be less profitable, perhaps unprofitable.

We implemented several improvements and bug fixes that should make the new software more practical. The slippage model addresses the run-away return and its effect on the market. And we also updated the data to use the S&P 500 futures data rather than the S&P 500 index. The change in data only slightly affects the results. All these together combine to change the optimal strategy.

Table 2 below shows the optimal parameters for the 2010 team and 2011 team strategies. Each row shows the optimal strategy under different circumstances. The values in brackets have been inferred from the policies; they are not parameters but are averages of values that are configured by the parameters. It is useful to compare them against parameters from other policies. First, we'll describe the data behind the rows and then we will analyze the table.

1. 2010 (1): the optimal strategy according to the previous team's final report (Chen, et al. 2010)
2. 2010 (2): the optimal strategy produced using the previous team's optimal parameters and the software as initially provided
3. 2010 (3): the optimal strategy using Dataset F, Spring 2010 strategy with our software changes
4. 2011: the optimal strategy using Dataset B, our Spring 2011 strategy

| Strategy | Days | Put | Call | Premium | Bear-Call Increment | Bull-Put Increment | Stop-Loss | Final TWR |
|---|---|---|---|---|---|---|---|---|
| **2010 (1)** | 42 | -15 | 5 | - | - | - | 20 | 711.30 |
| **2010 (2)** | 42 | -15 | 5 | [35.3] | - | - | 20 | 699.61 |
| **2010 (3)** | 39 | -35 | 5 | [30.3] | - | - | 15 | 84.31 |
| **2011** | 28 | [-32.8] | [19.4] | 20 | none | -55 | - | 9.05 |

**Table 2 Optimal parameters for the different strategies**

Rows 1 and 2 are an effective baseline against which we can compare our results. A 700x return on investment is extremely high. We would expect a lower return when changing these assumptions. The third row shows the best possible TWR under the spring 2010 strategy and the same assumptions we use for our spring 2011 strategy. This difference in TWR is mostly due to slippage.

The spring 2010 strategy actually does outperform our new iron condor strategy and the reason is in part from the put and call difference. Our strategy parameterizes over premium, using the same premium for both the short put and short call. Strike prices and their premiums are usually close to symmetric around the spot price, though not necessarily.

This is not the case for the policy with -35 put and 5 call. This seems to indicate that the -35 put option is rarely exercised and has a relatively high premium. The same is true for the call; it is rarely exercised, or has a sufficiently high premium to offset the losses. This indicates a bear market, which is true for large parts of 2007, 2008 and 2009 – the years over which these strategies were evaluated.

While our final TWR is roughly nine times less than that of the previous strategy, the TWR of 84.31 and 9.05 are based on assumptions that reasonably reflect the real-world trading market. A short strangle with stop-loss and an iron condor with symmetric short put/call options are different strategies and we expect one to outperform the other. However, we are pleased that the new TWR values are lower, indicating that the revised assumptions in the simulation truly do contribute the simulation realism.

We will now show several results from our analysis of the trading simulation. Additional results can be found in Appendix 10.

### 4.1.2 Days Before Expiration

One of the parameters in the trading simulation is the number of days before expiration to sell the strangle options. For this analysis we used Dataset A. There is a noticeable rise in average TWR on days 28, 25 and 24 before expiration. This is the Friday, Monday and Tuesday, respectively, 4 weeks before expiration. This rise in profit during this time is supported in the literature (1Option 2011).

Days 26 and 27 are missing, along with others, because we parameterize using calendar days, but no trading occurs on weekends and some holidays.

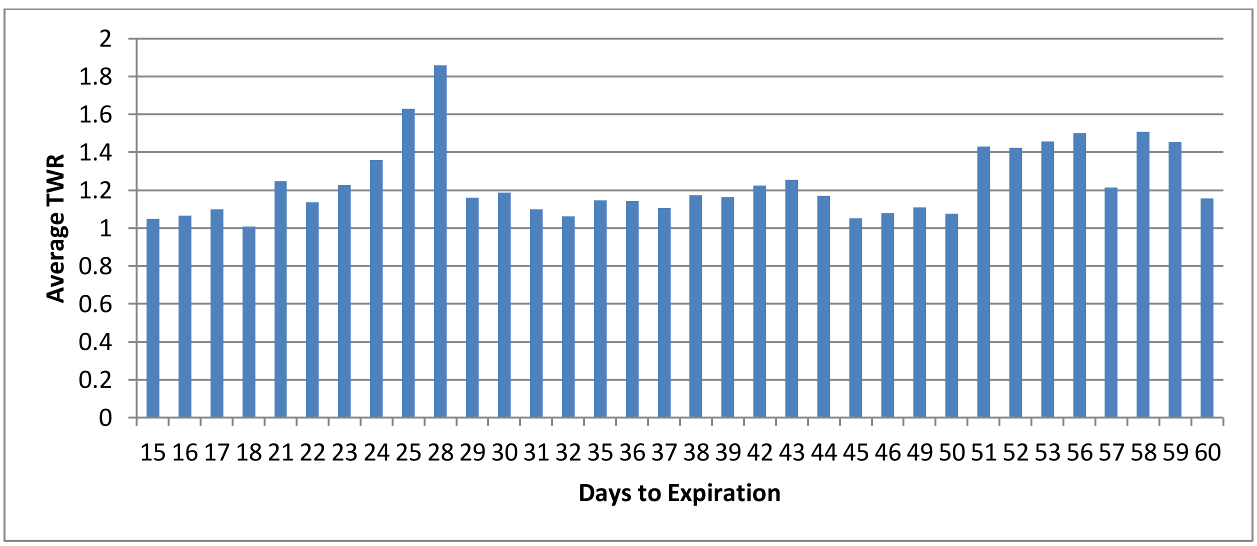


Figure 6 Average final TWR by days to expiration

While profit is considerably higher during days 24, 25 and 28 before expiration, the standard deviation is also higher. Standard deviation of return on day 28 is much higher than any other day. However, the chart below shows the Modified Sharpe Ratio, which takes the standard deviation of return into account. The Modified Sharpe Ratio for day 28 is lower than day 25, but still pretty good. Also, days 51 through 60 show consistent and strong returns.

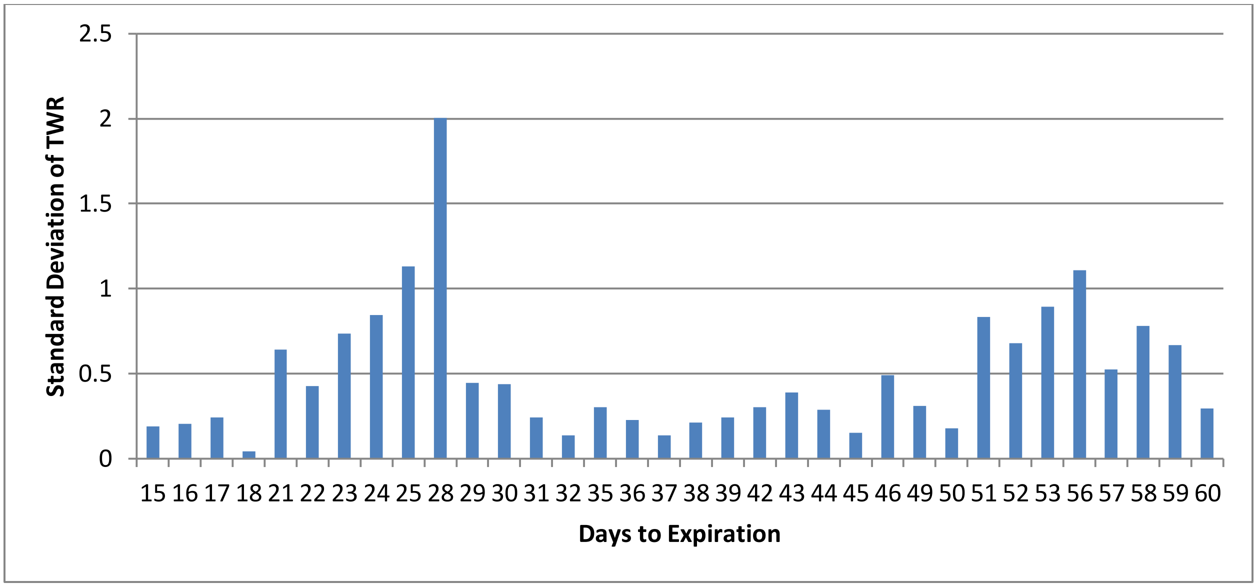


Figure 7 Standard deviation of final TWR by days to expiration

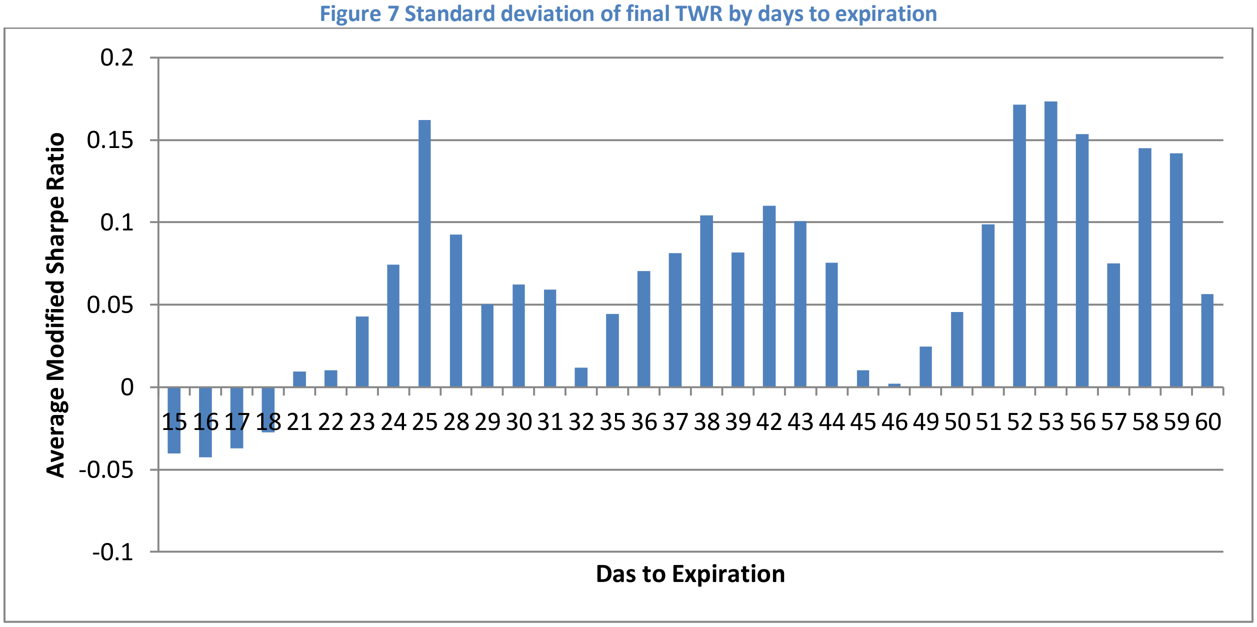


Figure 8 Average Modified Sharpe Ratio by days to expiration

### 4.1.3 Investment Fraction

Once an optimal policy is found, the speculator must determine the amount of capital to invest. The fraction to invest may be fixed or variable. We implemented a single variable strategy, Kelly's Criterion and 20 fixed strategies, 5% through 100%. After each policy is simulated, the software determines which of these 21 fractions is optimal, generating the greatest final TWR. Since each strategy chooses only one fraction as the optimal, we can break down the TWR by investment fraction.

The chart below shows the average TWR by investment fraction 5% through 100% and *K* for Kelly's criterion. The datasets used are Dataset C, Dataset D and Dataset E. These are identical, except that the first uses $1 million initial investment assuming no slippage. The second dataset also uses $1 million initial investment, but considers slippage. The final dataset uses $10 million initial investment with slippage.

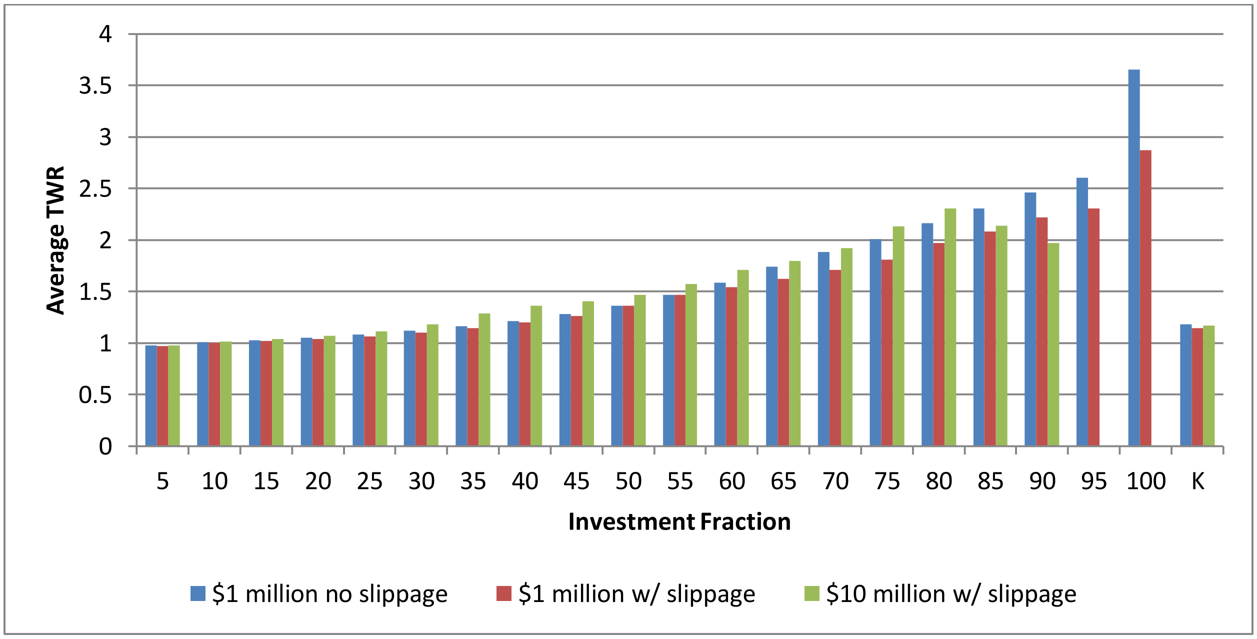


Figure 9 Average final TWR by investment fraction

For the blue series, assuming no slippage, when the strategy does very well, the investment allocation will be 100% and the final TWR will be high. For strategies that perform poorly, investing 5% is the best way to experience no risk and the ending value will be very close the initial value of the portfolio. For the red series, $1 million initial investment with slippage, the shape of the curve is very similar, except in this case the returns are slightly lower. The best-performing policies still pick 100% as the optimal investment fraction, but since there is some slippage, the return is lower. This is likely due to the permanent slippage, as explained in Appendix 8.

The green series, having $10 million initial investment with slippage, shows a more significant change from the blue series. There is now no strategy that chooses 95% or 100% allocation as the optimal investment, even if a policy consistently wins. The reason is that an investment of $10 million is a market maker in the small options market. Several wins and doubling the investment means a much larger impact on the market. This seems to indicate that the best performing strategies choose 80% as the optimal investment fraction.

The following chart may be more informative. The horizontal axis is the final TWR for the policy and the vertical axis is the optimal investment fraction. This shows that without slippage, all the highest performing policies have investment fractions of 100%. The same policy with same initial investment amount but with slippage will have lower final TWR, but still use 100% investment fraction. A higher initial investment with slippage, however, finds lower TWR and invests only up to 80% for those optimal policies.

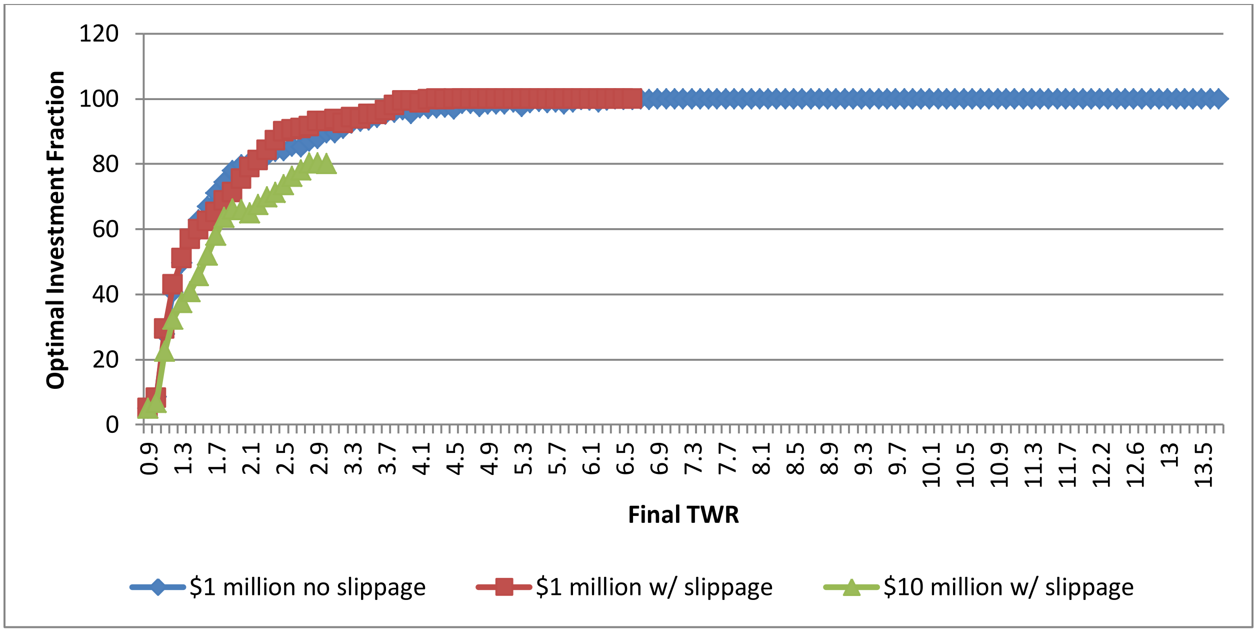


Figure 10 Optimal investment fraction by final TWR

The following table shows that the optimal policies for all three datasets are very similar. The top two policies for $10 million investment with slippage are in the top three strategies for the other datasets.

| Days | Premium | Bear-Call Increment | Bull-Put Increment | Optimal Fraction | Final TWR | % Wins | Avg Win | Investment | Slippage |
|---|---|---|---|---|---|---|---|---|---|
| 28 | 19 | 100 | 55 | 100 | 13.94 | 71.43 | 21.14 | $1m | No |
| 28 | 20 | 100 | 55 | 100 | 13.92 | 71.43 | 21.09 | $1m | No |
| 28 | 20 | 100 | 50 | 100 | 13.9 | 71.43 | 20.24 | $1m | No |
| 28 | 20 | 100 | 55 | 100 | 6.56 | 71.43 | 21.09 | $1m | Yes |
| 28 | 19 | 100 | 55 | 100 | 6.55 | 71.43 | 21.14 | $1m | Yes |
| 28 | 20 | 100 | 50 | 100 | 6.53 | 71.43 | 20.24 | $1m | Yes |
| 28 | 19 | 100 | 55 | 80 | 3 | 71.43 | 21.14 | $10m | Yes |
| 28 | 20 | 100 | 55 | 80 | 3 | 71.43 | 21.09 | $10m | Yes |

Figure 11 Table of optimal policies for the 3 slippage datasets, showing the optimal policies are constant

## 4.2 Prediction Model Results

In the prediction model, we solve for the options strike price as described in 3.3 and in detail in Appendix 9, using the Black-Scholes-Merton model as a function of premium, spot price, risk-free interest rate, days to expiration and asset volatility. We then compute the expected profit which requires the annualized expected return on asset price and the bear-call and bull-put strike price spreads. To validate our initial prediction model results we first used the actual annualized returns on asset price for a given trade (from the asset price at *N* days before expiration to the spot price at expiration). This implementation of using the actual annualized returns on asset price however should not be considered a valid prediction model approach, since in general the asset price at expiration will not be known.

Given the assumptions in 2.4.7, calculating the expected value of a particular strategy can be done exactly using the analytical prediction model. However, we must also verify that the results of the prediction model are equivalent to the actual results from historical data. We will show that there are some parameters that cause the prediction model results to differ significantly from the actual results.

### 4.2.1 Difference in Profit

We used the following two metrics to compare the performance of the prediction model against the actual results:

1. *Difference Between Expected Profit and Realized Profit*: absolute value of the difference between (1) the Prediction Model's expected value of the profit and (2) the prediction model's realized profit computed based on the options strike prices and the stop price at expiration.
2. *Difference Between Realized Profit and Simulation Profit*: absolute value of the difference between (2) the prediction model's realized profit computed based on the options strike prices and the stop price at expiration and (3) the profit determined by running the same strategy parameters through the trading simulation.

A large value of metric 1 means that market conditions created a distribution too wide to accurately predict the realized profit. Metric 2 measures the effect of using Black-Scholes to calculate strike prices versus using market data.

Plotting the average difference in profit by the number of days until expiration shows there is a fairly consistent separation between the prediction model's expected profit and realized profit and also between the prediction model's realized profit and the trading simulation's profit, however the number of days does not appear to significantly affect that separation. This would indicate that the number of days until expiration can be changed without affecting the accuracy of the prediction model.

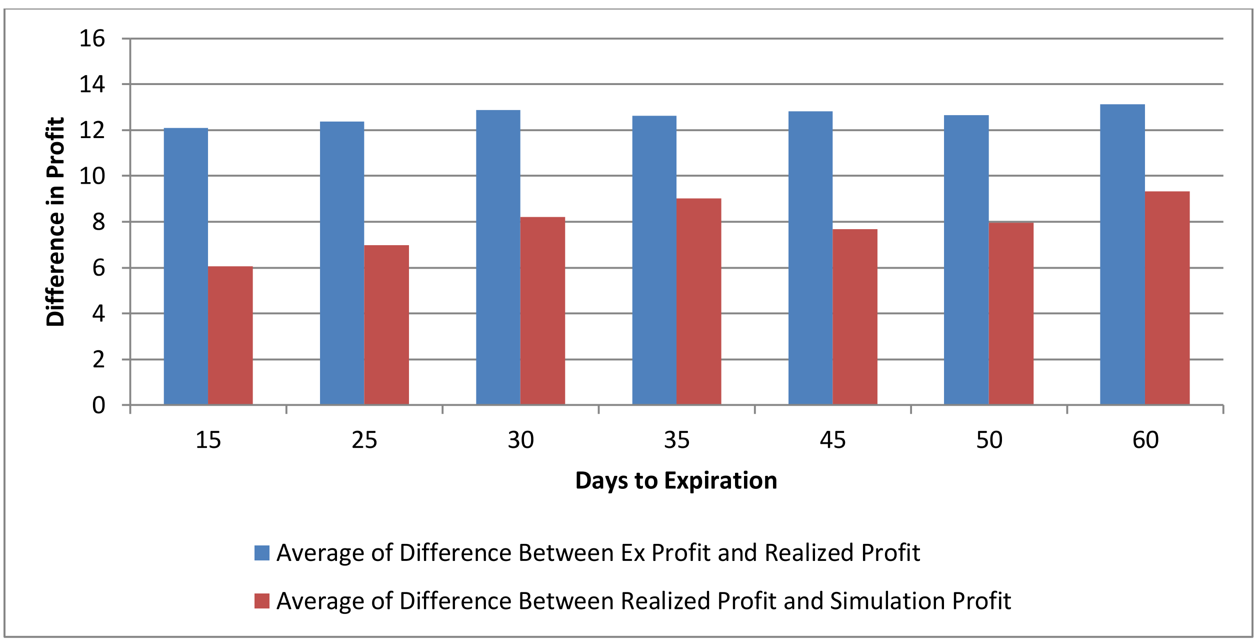


**Figure 12 Plot showing how days to expiration affects the prediction of the model**

Unlike days to expiration, premium does appear to affect the difference between expected and realized profit. For a premium of 50 or 60, both metrics 1 and 2 are lower than they are at a premium of 10 or 100.

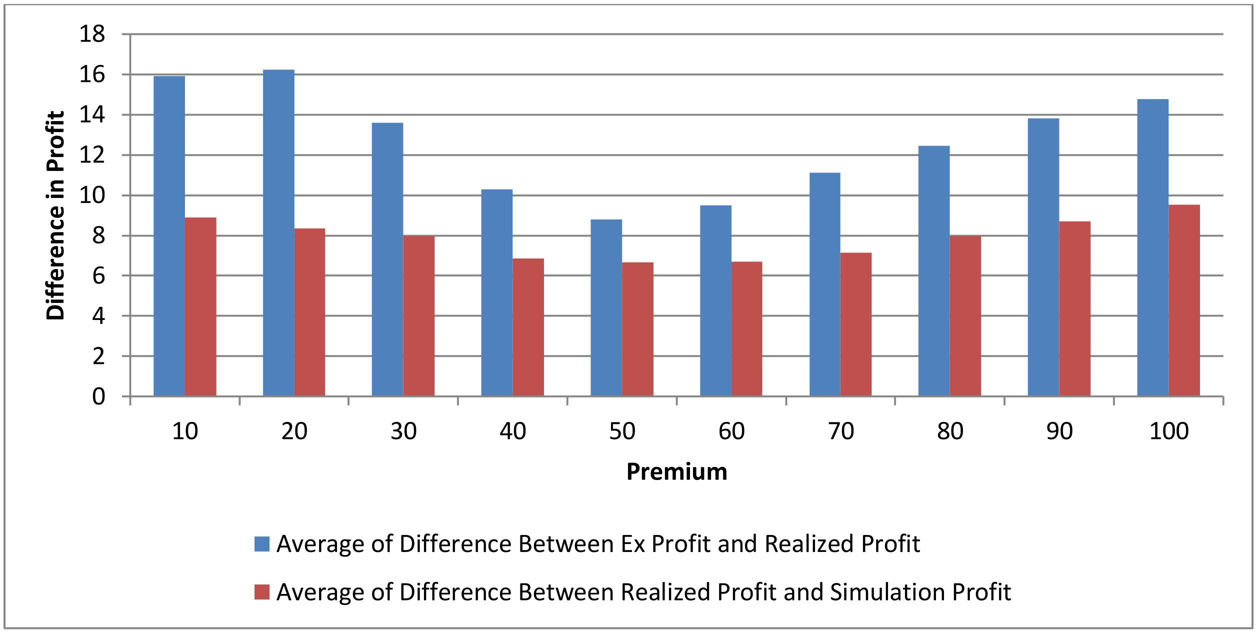


**Figure 13 Plot showing how premium affects the prediction of the model**

### 4.2.2 Difference in Strike Price

When comparing results for the prediction model's expected profit and realized profit against the simulation's profit we are essentially comparing how the strike price for an option is determined: either using Black-Scholes-Merton or market data. Both approaches, and in particular Black-Scholes-Merton, use the initial stock price when an option is written and the risk-free rate of interest as parameters. However, without a deeper understanding of financial markets and trends, we assumed the effect of either parameter on determining an options strike price between the two approaches was equivalent and canceled out.

In the prediction model, asset volatility is computed using historic data as explained in Appendix 9. The following chart shows implied volatility from the options data versus volatility computed from the previous 180-days over our 3-year period of interest from 2007-2009. The chart demonstrates that if implied volatility data is not available, the computed volatility appears to be a sufficient replacement and follows the same trends. However the large jumps in implied volatility may be causes of differences between strike prices determined from Black-Scholes and market data.

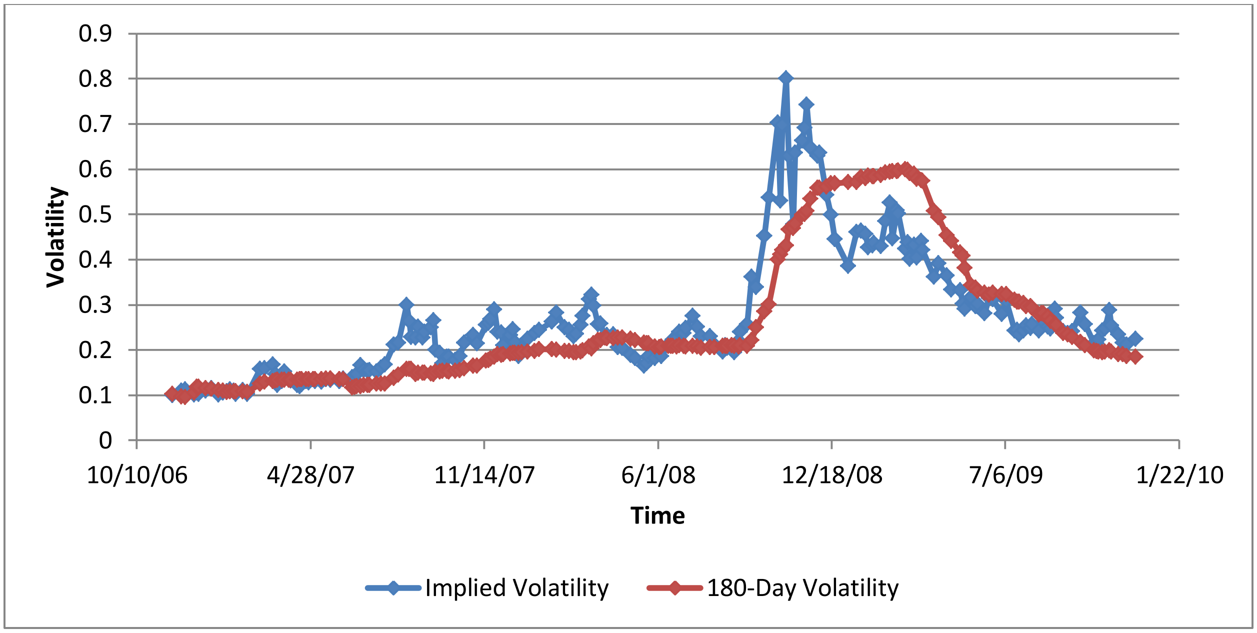


Figure 14 A comparison of implied and computed volatility

The other two parameters that affect strike prices are premium and days to expiration. The following charts show that as a function of days to expiration, the difference between put and call option strike prices generally increases as the days to expiration increase. When closer to expiration, the predicted and actual strike prices are much closer because the market is not likely to move significantly in the short time remaining.

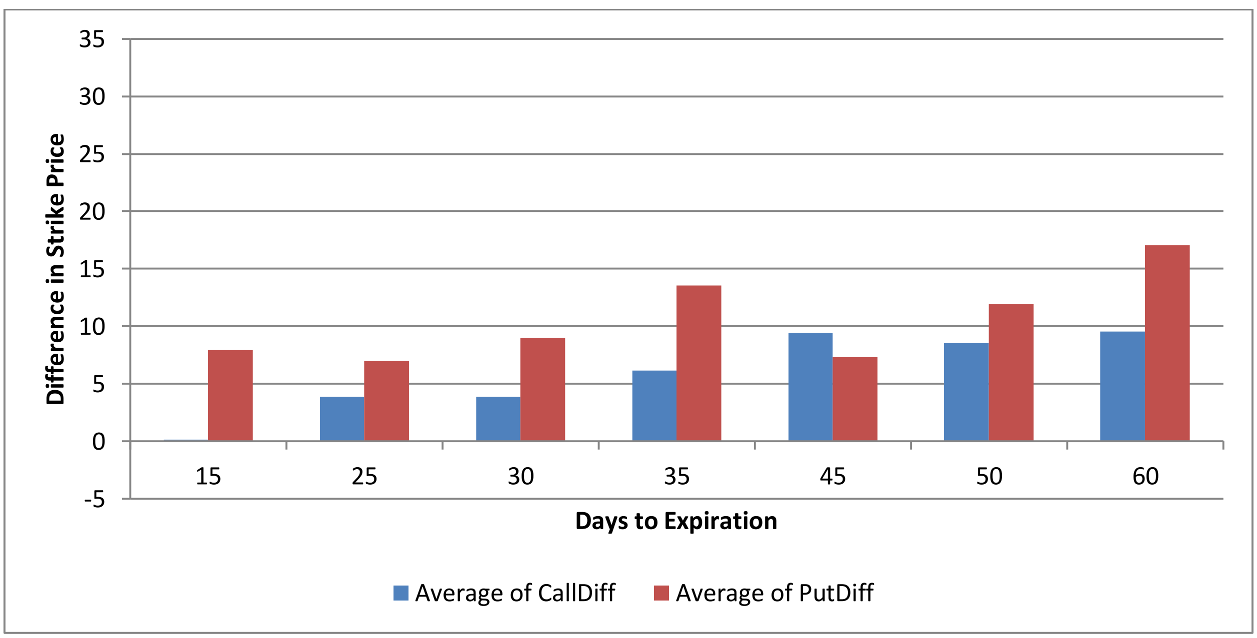


Figure 15 Difference between Black-Scholes strike prices and market data strike prices as a function of days to expiration

As a function of premium, the difference between call options strike prices starts relatively high then decreases as premium increases. The difference between put options strike prices also starts relatively high and decreases as premium increases with a minimum around a premium of 50-60 points, however the difference then increases relatively high again as the premium increases to a premium of 100 points.

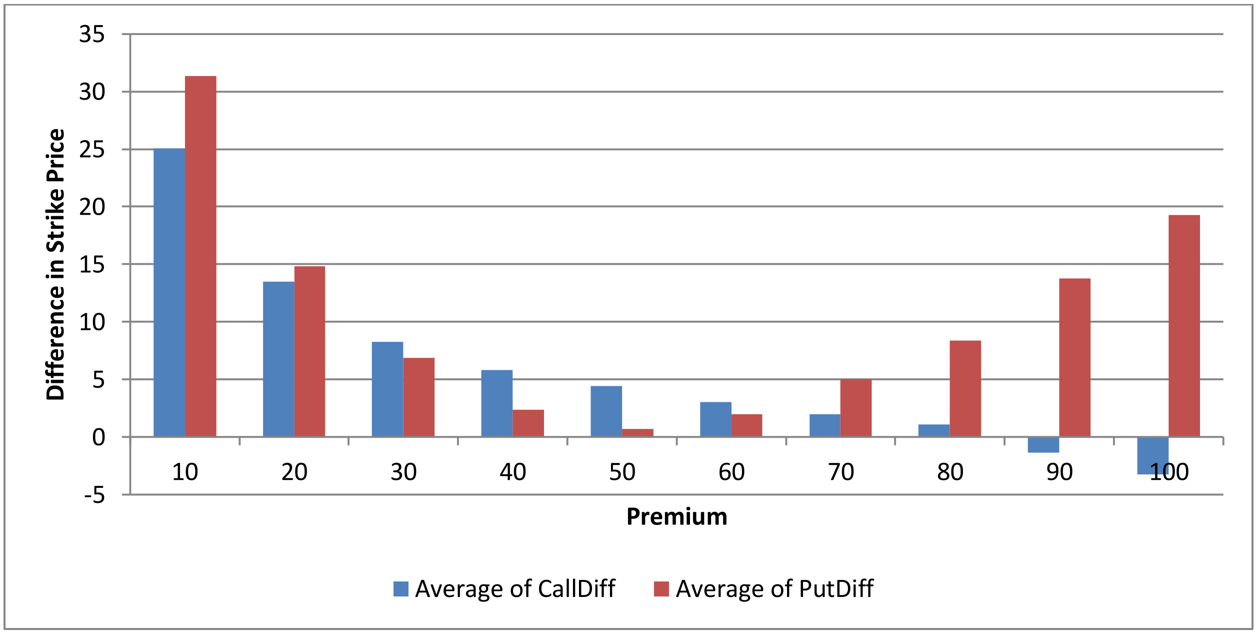


Figure 16 Difference between Black-Scholes strike prices and market data strike prices as a function of premium

This helps explain both Figure 12 and Figure 13. Since we parameterize by premium, a prediction model strategy and a simulated strategy may have the same premium, but different strike prices. And as these charts show, the strike prices may be significantly different. For example, with a 10 point premium, the Black-Scholes-Merton model determines a put and call option with strike prices that are on average at least 25 points above the same put and call options found in the options data with 10 points premium. This difference in strike prices will significantly affect the difference between the profit from the prediction model and the simulation.

### 4.2.3 Market Conditions

Observing that the prediction model results for a desired premium of 60 points has the minimum difference between the expected profit and the realized profit, we now focus on the bear-call and bull-put spread increment by whether the market is going up or down. We consider the market going up when the actual annualized return on asset price is positive and consider the market going down when the actual annualized return on asset price is negative. Recall the actual annualized return on asset price is based on the asset price at the contract date and the asset price at expiration. During the specific trading days evaluated in the years 2007-2009, the market was up roughly 58.3% of the time and down roughly 41.7% of the time.

The following charts show the prediction model's average expected profit in BLUE, the prediction model's average realized profit in RED and the trading simulation's average profit in GREEN by the bear-call and bull-put strike increment for a premium of 60 points.

When the market is up, all three models achieve their maximum profit using a bear-call strike price 10 points over the call option strike price and a bull-put strike price 100 points below the put strike price. This is an obvious strategy when the market is going up – buy a low bear-call as insurance on the call, but don't bother buying insurance on the put.

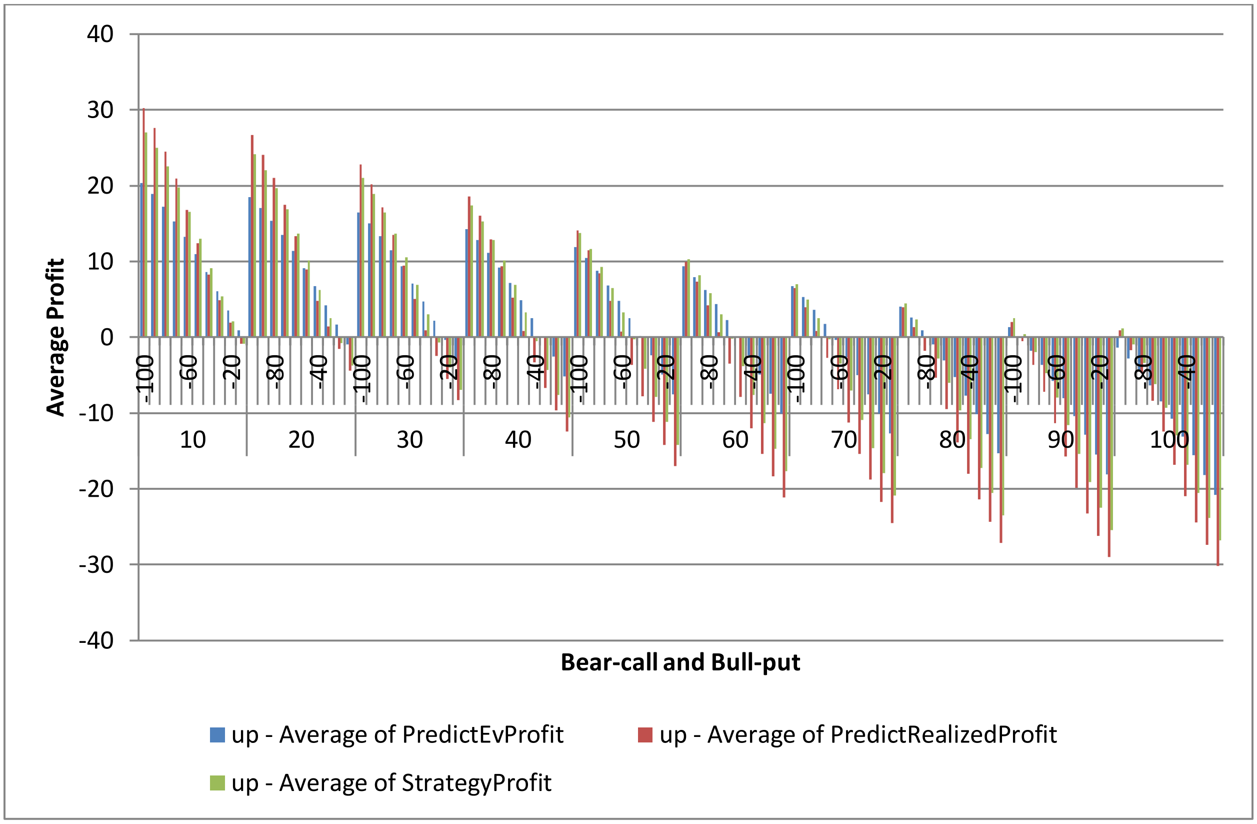


Figure 17 Performance of the prediction model by bear-call and bull-put increments in a bull market

When the market is down, all three models achieve their maximum profit using a bear-call strike price 100 points over the call option strike price and a bull-put strike price 10 points below the put strike price. This indicates buying a bull-put as insurance on the put and no bear-call when the market is down.

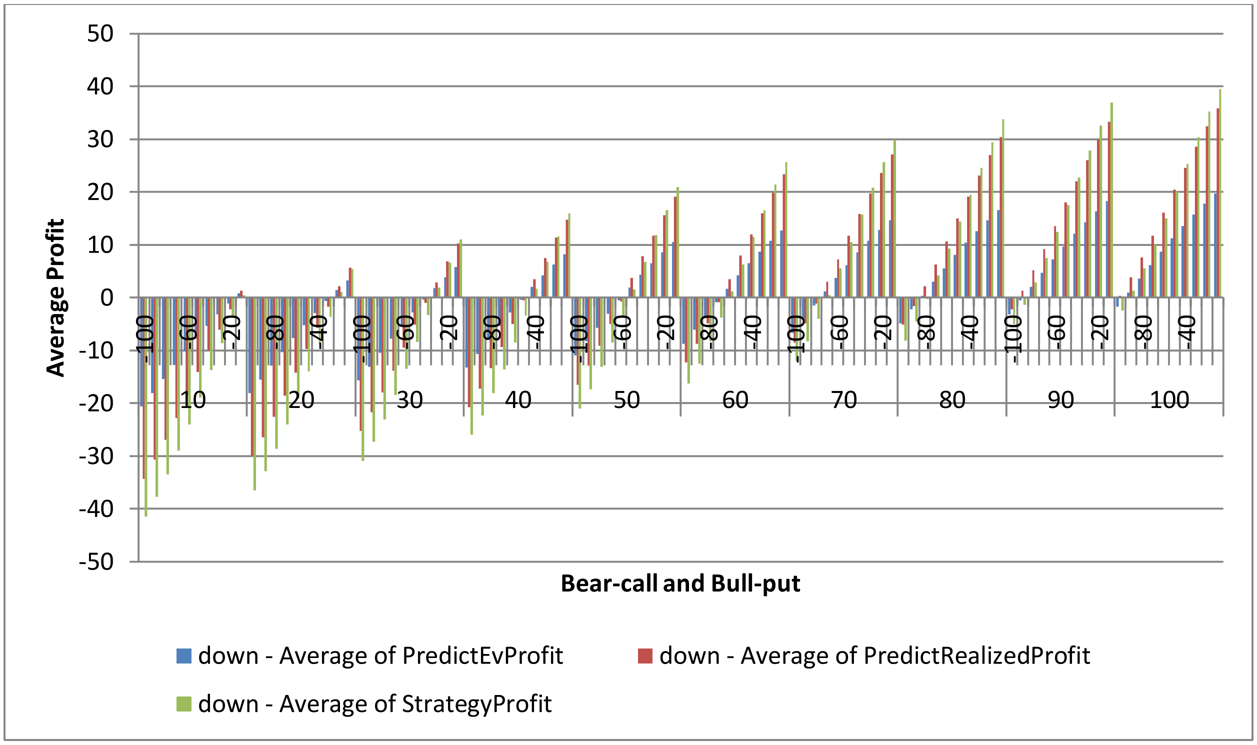


Figure 18 Performance of the prediction model by bear-call and bull-put increments in a bear market

### 4.2.4 Using Estimated Annualized Return on Asset Price

Next our team evaluated the prediction model by estimating the annualized return on asset price from historical pricing data. In the following results, our team fixed premium at 60 points which as shown earlier had the minimum difference between the prediction model's expected profit, realized profit and the simulation's profits when using the actual annualize return. We then computed the estimated annualized return as described in Appendix 9 for the previous 90, 135, 180, 225, 270, 315, and 360 days before a given contract date.

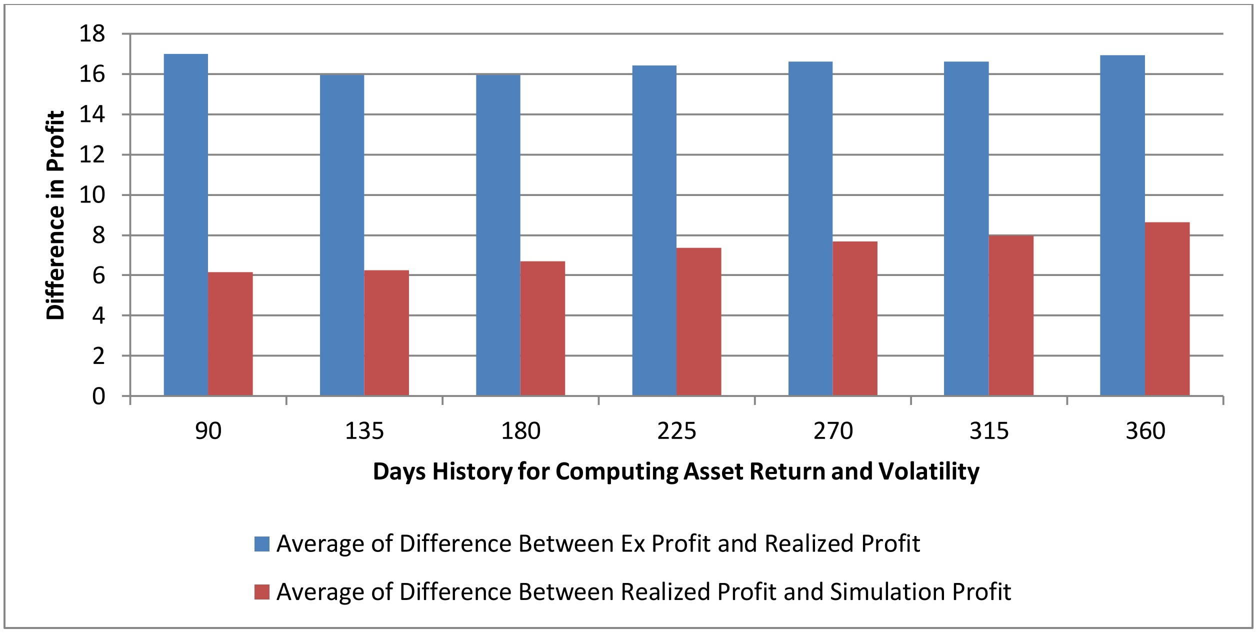


**Figure 19 Difference in profit as a function of the number of days used to compute asset return and volatility**

In the chart above, RED is the difference between the prediction model's realized profit and the simulation profit. The RED series increases as the number of historical days increases due to increasing errors in the computed volatility over the previous *N* days. While these are not necessarily errors, volatility computed using the past 360 days does not necessarily reflect the current implied volatility very well. In the chart below, volatility computed using the past 360 days for the last quarter in 2009 includes the market crash in the last quarter of 2008. Therefore, while the implied volatility is low, the computed volatility is high. The difference in volatility estimate causes the relationship between premium and strike price computed using Black-Scholes to be significantly different then when using the options data to determine strike prices from premium. Once the options strike prices are different, the prediction model's realized profit and the simulation's profit diverge.

The series in BLUE above shows the difference between the prediction model's expected profit and realized profit using the previous *N* days to compute asset return and volatility. From this chart, there does not appear to be any interval of historic price data that more accurately forecasts future asset returns better than any other.

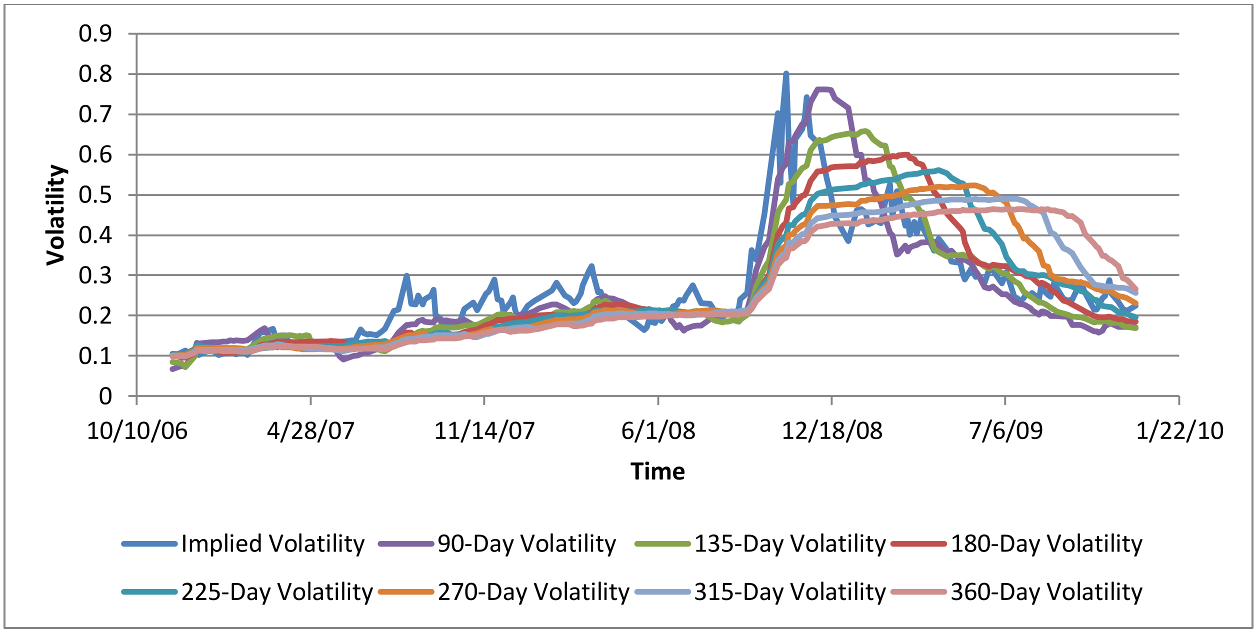


**Figure 20 Estimate of index volatility as a function of the number of historical days used to compute it**

The next chart shows the annualized rate of return computed for the prediction model. In blue is the actual annualized rate of return for trades 15 days before expiration. That is, the return of the expiration price over the index price 15 days before expiration, scaled for an annual rate. This shows that none of the computed returns, using historical data, come close to the actual rate of return. This is especially true for the 30x return rate achieved from March 5, 2009 to expiration on March 20, 2009 when the E-Mini S&P 500 Futures jumped from 686 to 789.5. Also notice that all estimated return rates were negative for that contract date.

Because the market is random process we expect an unpredictable rate of return, but the historical estimators as computed in Appendix 9 are significantly off in most cases. A better predictor of market return would give the prediction model greater accuracy and utility to potential investors.

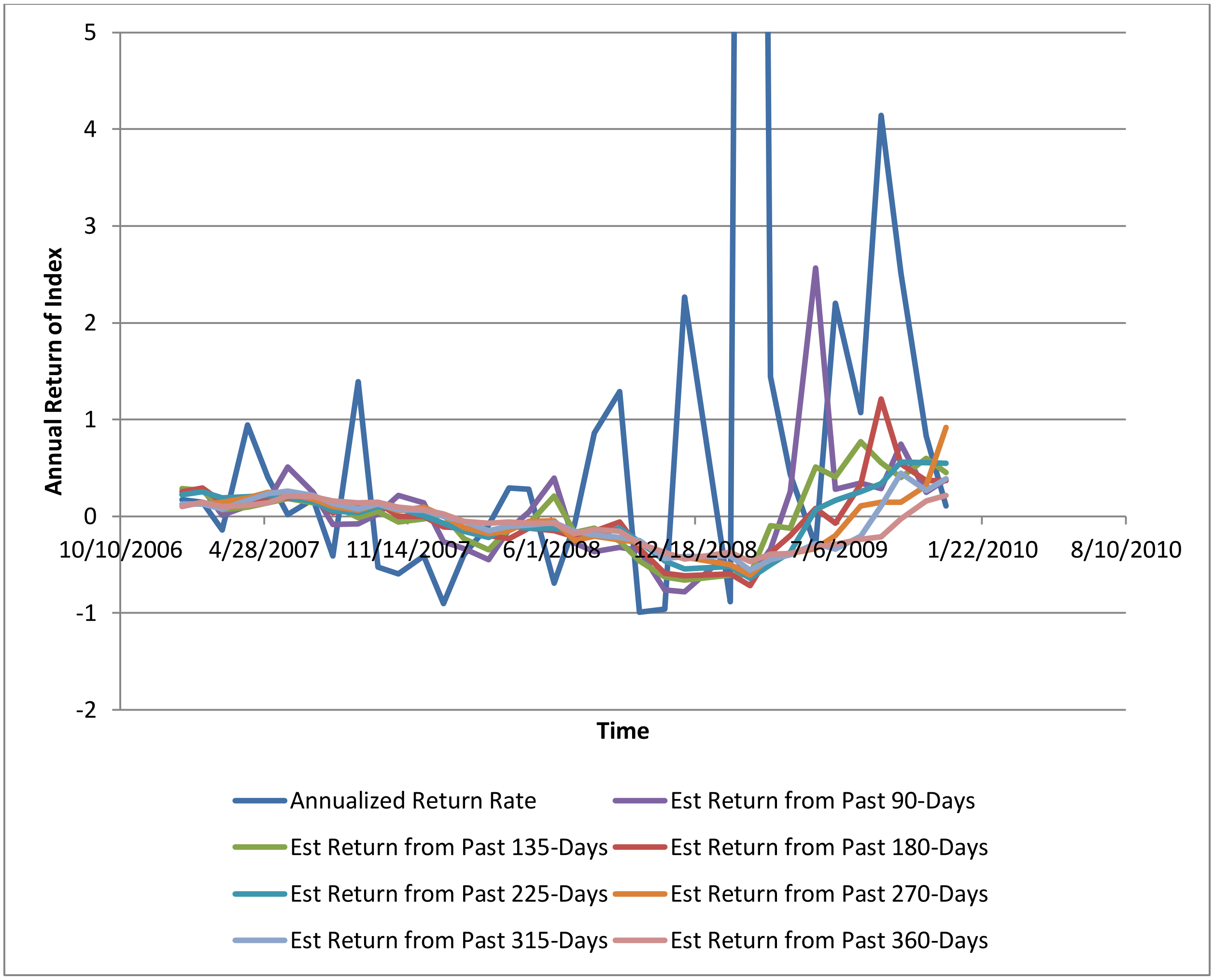


**Figure 21 Annualized return of the index as a function of number of days used to compute it**

# 5 Conclusion

The main goal of this project was to analyze options trading strategies. The two ways we chose to do this were simulation and explicit calculation of the expected value. We completed both goals, first extending the simulation software, improving assumptions and adding features, then developing and implementing a theoretical prediction model.

## 5.1 Recommendation

It is difficult to recommend a specific combination of parameters as the optimal trading policy. Instead, our analysis in section 4.1 showed the optimal ranges for several parameters. Low values of premium for the short strangle options are generally better. Investing approximately four weeks before expiration results in greater return. However, it is hard to draw conclusions on the bear-call and bull-put increments from the data. The values of these options are highly dependent on the direction of the market. This may also be the case for the premium and days to expiration, but the link is less obvious. If a market is bearish, a trader should buy the bull-put very close and no bear-call at all. If the market is bullish, the opposite is true.

The prediction model was not designed to recommend a best possible trading policy for all scenarios. It is implemented as a predictor of returns given a set of specific parameters. Unfortunately, as we showed in 4.2.1, parameter values affect the accuracy of the prediction model. Days to expiration does not appear to significantly affect results, but premium does. Perhaps the most important parameter is the annual return of the market. Without an accurate estimate of the return of the index, the expected profit from the prediction model will have significant error. Therefore, we recommend further investigation and research into the prediction model to determine a more accurate method for forecasting returns.

## 5.2 Future Work

### 5.2.1 American Options

All of our work was done with European options, but there are many different option styles, such as American. The main difference between an American and European option is that an American option can be exercised at any point between the initial trade and the expiration date. There is no simple formula for pricing the American option as there is for the European. But it can be simulated.

### 5.2.2 Identify Best Strategies for Specific Market Conditions

We did not have the time to do the appropriate analysis and find the optimal trading approach for each type of market, other than the simplistic up or down trend. An adaptive trading strategy would use the market history and attempt to determine the current trend and adjust the strategy parameters based on the current market conditions.

### 5.2.3 Trade for Specific Months

The current approach is to trade for each month in the year. It would be straightforward to add a parameter to the software to specify which months to trade. This would allow analysis on which months were especially profitable for trading.

### 5.2.4 Most Recent E-Mini S&P 500 Options Data

Our E-Mini data included the years from 1999 through 2009, but was very sparse before 2004. It would be useful to have more up-to-date data and even older data to fill in the gaps. Additional data allows analysis over a wider range of market conditions. Most of our analysis was over 2007-2009, which includes a market crash, which is not favorable to a strangle trading strategy.

Alternatively, the E-Mini S&P 500 data could be replaced by another index, such as the E-Mini Dow or E-Mini NASDAQ-100. There is nothing special about the S&P 500 that limits our software to that index, except that it is especially suited to options writing. A survey of possible indices and an analysis of strategies across other indices would be a useful tool for a potential investor.

### 5.2.5 Forecasting Models

As indicated in Appendix 9 the prediction model uses an annualized return on the asset price as part of the expected value computation. The simplest method is to take the actual return of expiration price over the spot price on the initial trading day. This method gives an expected profit closest to the actual profit. But since the trader only has historical data, the return should be estimated using historical data.

Our team tried a simple first-order approximation method of computing the estimated annualized return over the past *N* days, but this approach does not follow the trend of future market conditions very well. More sophisticated models of predicting future returns should result in a more effective prediction model.

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# 7 Appendix – Parameters for Analysis

To help the reader repeat our results, we detail here the parameters for the simulations used to generate our analyses.

## 7.1 Dataset A

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $1,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: no

Using the Spring 2011 Strategy
Premium: 10-100, increments of 2
Bear-Call/Bull-Put: 5-100, increments of 5
Include No Spread: yes
Maximum volatility: none
Total number of strategies: 689,724

## 7.2 Dataset B

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $1,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: yes

Using the Spring 2011 Strategy
Premium: 10-100, increments of 2
Bear-Call/Bull-Put: 5-100, increments of 5
Include No Spread: yes
Maximum volatility: none
Total number of strategies: 689,724

## 7.3 Dataset C

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $1,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: no

Using the Spring 2011 Strategy
Premium: 5-25, increments of 1
Bear-Call/Bull-Put: 50-100, increments of 5
Include No Spread: no
Maximum volatility: none
Total number of strategies: 86,394

## 7.4 Dataset D

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $1,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: yes

Using the Spring 2011 Strategy
Premium: 5-25, increments of 1
Bear-Call/Bull-Put: 50-100, increments of 5
Include No Spread: no
Maximum volatility: none
Total number of strategies: 86,394

## 7.5 Dataset E

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $10,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: yes

Using the Spring 2011 Strategy
Premium: 5-25, increments of 1
Bear-Call/Bull-Put: 50-100, increments of 5
Include No Spread: no
Maximum volatility: none
Total number of strategies: 86,394

## 7.6 Dataset F

Years: 2007-2009
Trading Days: 15-60
Investment Amount: $10,000,000

Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: yes

Using the Spring 2010 Strategy
Put Option Range: -100 to -5, increments of 5
Call Option Range: 5 to 100, increments of 5
Stop Loss: 5 to 45, increments of 5 and no stop loss
Maximum volatility: 20, 30, 40, none
Total number of strategies: 404,720

## 7.7 Dataset G

Years: 2004
Trading Days: 15-60
Investment Amount: $10,000,000
Required Margin: $5,000
Ruin Fraction: 0.50
Slippage: no

Using the Spring 2011 Strategy
Premium: 6-50, increments of 2
Bear-Call/Bull-Put: 5-100, increments of 5
Include No Spread: no
Maximum volatility: none
Total number of strategies: 312,800

# 8 Appendix – Slippage Model

Slippage is primarily a function of trade size. Our team found a model for slippage and incorporated it into the calculation of TWR. This affects the TWR of each fraction of allocation and causes the 100% investment to become non-optimal for very large investments.

Our model for slippage is a discrete-time model where randomness in the market is modeled using a binomial model (Bakstien n.d.). When buying, the prices will slip somewhat simply because there is a delay in any market order. This permanent slippage is due to the volatility of the market. Hence

$$\dot{S}_{t+1} = \begin{cases} e^{\sigma/\sqrt{\Delta t}} \cdot S_t & if\ buying \\ e^{-\sigma/\sqrt{\Delta t}} \cdot S_t & if\ selling \end{cases}$$

where σ is volatility and Δt is a single time step, or one day in our case. We can determine volatility either by calculating it from historical data, or using the implied volatility in the data. We used the implied volatility of S&P 500 index.

The second piece in any trade is how the market reacts to the size of the trade. If a trade is large, the transaction cost will be greater, lowering the profit of the trade. The size of the trade is relative to the depth of the market, or market volume. We can model the instant reaction of the market to a trade by using the following (Bakstien n.d.):

$$\ddot{S}_{t+1} = S_t \cdot e^{\lambda(H_{t+1} - H_t)}$$

where $H_{t+1} - H_t$ is the size of the trade. λ is known as the liquidity parameter, measuring the reaction of the market to any trade. Liquidity is formulated as

$$\lambda = \frac{\Delta S / S}{\Delta H}$$

where ΔS/S is the relative change in price and ΔH is the total daily volume (Bakstien n.d.). Rather than allowing liquidity to be formulated as ΔS/ΔH, this formulation gives a unitless change in price in terms of the daily volume. This term amplifies the permanent slippage by an amount proportional to the size of the trade.

Putting both of these parts together, we have

$$S_{t+1} = S_t e^{\alpha\sigma/\sqrt{\Delta t}} e^{\lambda(H_{t+1} - H_t)}$$

where α is 1 if we are buying and -1 if we are selling.

# 9 Appendix – Prediction Model

## 9.1 Finding Strike Price Using Black-Scholes-Merton

The first step in our prediction model is to find the option strike price by premium using the Black-Scholes pricing formulas for European put and call options:

$$C = S_0 \cdot \Phi(d_1(K_c)) - K_c \cdot e^{-rT} \cdot \Phi(d_2(K_c))$$

$$P = K_p \cdot e^{-rT} \cdot \Phi\big(-d_2(K_p)\big) - S_0 \cdot \Phi\big(-d_1(K_p)\big)$$

$$d_1(K) = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2(K) = d_1(K) - \sigma\sqrt{T}$$

With asset price at time zero $S_0$, option strike price $K$, continuously compounded risk-free rate of interest $r$, the asset price volatility $\sigma$, the time to option maturity $T$, and where $\Phi(x)$ is the cumulative probability distribution function for the standard normal distribution:

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t)dt = \tfrac{1}{2}\left[1 + \text{erf}\left(\tfrac{x}{\sqrt{2}}\right)\right]$$

$$\varphi(x) = \tfrac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

There is no easy way to invert the put or call formulae so instead we solve for strike price numerically using Newton's method. Newton's Method is an iterative technique that constructs a sequence on the strike price $K_n$ that in general converges quadratically towards $K$. The sequence is defined as:

$$K_{n+1} = K_n - \frac{f(K_n)}{\dot{f}(K_n)}$$

Where $f(K)$ is the pricing function for a European put or call option minus the premium and set equal to zero and $\dot{f}(K)$ is the derivative of $f(K)$.

More specifically, for the call option, we define the function

$$f_c(K) = S_0 \cdot \Phi\big(d_1(K)\big) - K \cdot e^{-rT} \cdot \Phi\big(d_2(K)\big) - C = 0$$

With derivative

$$\dot{f_c}(K) = \tfrac{-S_0 \cdot \varphi(d_1(K))}{K\sigma\sqrt{T}} - e^{-rT}\left[\Phi(d_2(K)) - \tfrac{\varphi(d_2(K))}{\sigma\sqrt{T}}\right]$$

For the put option, we define the function

$$f_p(K) = K \cdot e^{-rT} \cdot \Phi(-d_2(K)) - S_0 \cdot \Phi(-d_1(K)) - P = 0$$

With derivative

$$\dot{f}_p(K) = e^{-rT}\left[\Phi(-d_2(K)) + \frac{\varphi(-d_2(K))}{\sigma\sqrt{T}}\right] - \frac{S_0 \cdot \varphi(-d_1(K))}{K\sigma\sqrt{T}}$$

When using Newton's Method, our initial guess is $K_0 = S_0$ and we exit the sequence when

$$|K_{n+1} - K_n| < \$0.01$$

## 9.2 Computing Expected Profit

Once we have solved for either the put or call option strike price, we can then compute the expected value of the profit of our option. First we assume the stochastic process governing our asset price is an Itô Process, and then the value of the asset price at some time $T$ in the future follows a lognormal distribution. Next we can define a random variable $Y$ as normally distributed with mean $\mu_y$ and standard deviation $\sigma_y$, so that:

$$Y = \ln(S_T) \sim N[\mu_y, \sigma_y^2]$$

$$\mu_y = \ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T$$

$$\sigma_y^2 = \sigma^2 T$$

With the asset price at time zero $S_0$, the annual asset price volatility $\sigma$, and the time to option maturity $T$, are as given above, and the annual expected return on asset price is $\mu$.

Second, we define the profit or payoff function of our options as:

$$h(S_T) = P - P_{ic} + g(S_T)$$

Where $P$ is the short option premium, $P_{ic}$ is the long (bear-call or bull-put) option premium and $g(S_T)$ is the options intrinsic value function.

The intrinsic value of a call option to an options writer is given by:

$$g_c(S_T) = \begin{cases} K - K_{bc}, & S_T > K_{bc} > K \\ K - S_T, & K_{bc} > S_T > K \\ 0, & K_{bc} > K > S_T \end{cases}$$

Where the call strike price $K$ is determined using Newton's Method above and the strike price of the options writer's bear call spread $K_{bc} = K + B_c$ is specified as a parameter $B_c$.

The intrinsic value of a put option to an options writer is given by:

$$g_p(S_T) = \begin{cases} K_{bp} - K, & S_T < K_{bp} < K \\ S_T - K, & K_{bp} < S_T < K \\ 0, & K_{bp} < K < S_T \end{cases}$$

Where again the put strike price $K$ is determined using Newton's Method above and the strike price of the options writer's bull put spread $K_{bp} = K + B_p$ is specified as a parameter $B_p$.

The expected value of the profit of an option is the inner product of *Y* and the payoff function $h(S_T)$ where $S_T = e^Y$:

$$E[h(S_T)] = \int_{-\infty}^{\infty} \varphi(y, \mu_y, \sigma_y) \cdot h(e^y) dy$$

Since integrating from negative infinity to positive infinity is not technically feasible and also *Y* is normally distributed, we can instead approximate the expected value with arbitrary accuracy by integrating symmetrically about the mean of *Y* by $n\sigma_y$:

$$E[h(S_T)] \approx \int_{\mu_y - n\sigma_y}^{\mu_y + n\sigma_y} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot h(e^y) \cdot dy$$

In our model we set *n* = 6 and computed the expected value numerically using Simpson's Rule.

Additionally we estimate both the annual asset price volatility $\sigma$ and the annual expected return on asset price $\mu$ in the following manor using historical asset price data $m$ days before day zero where the value of $m$ is a user specified input parameter (usually between 90 and 180):

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$\hat{\mu} = \left(1 + \frac{1}{m} \sum_{i=-m}^{0} u_i\right)^{252}$$

$$\frac{\hat{\sigma}}{\sqrt{252}} = \sqrt{\frac{1}{m-1} \sum_{i=-m}^{0} u_i^2 - \frac{1}{m(m-1)} \left(\sum_{i=-m}^{0} u_i\right)^2}$$

# 10 Appendix – Additional Simulation Analysis

## 10.1 Premium

In order to analyze the effect of premium on the final TWR, we used Dataset A. It appears policies with low premium generally provide the best return. These are the policies which have the put and call far away from the spot price, meaning the options have less chance of expiring in the money. Premium above 35 points has little effect on final TWR, except once premium reaches 80 points or higher. Policies with premiums this high will have strike prices very close to the spot price or even in the money at the time of the trade.

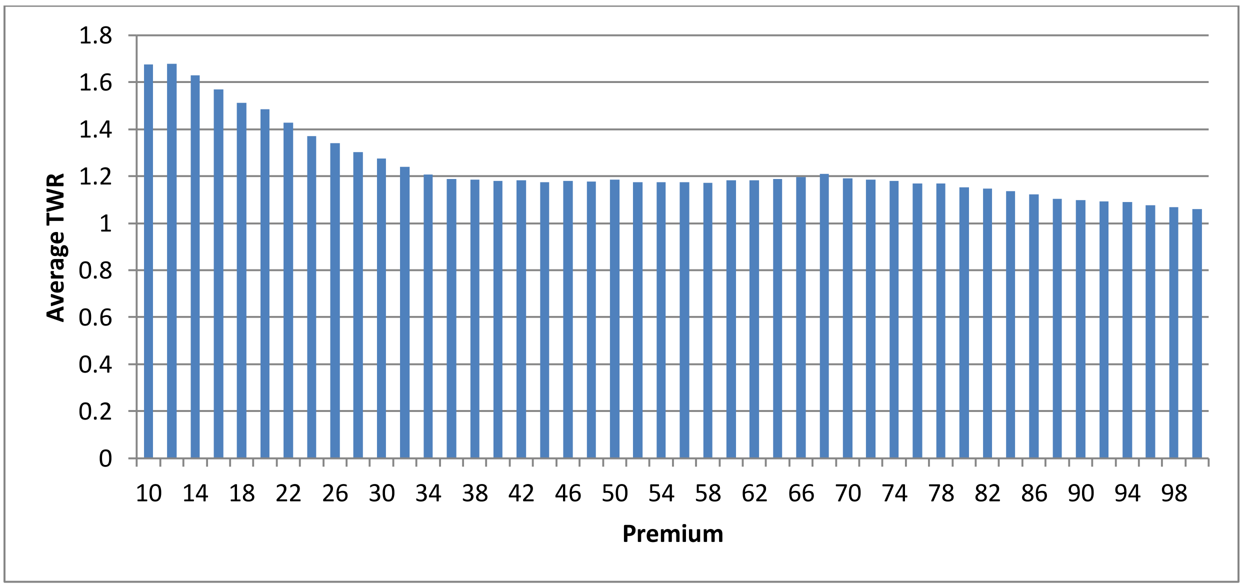


Figure 22 Average final TWR by premium

If we restrict the results to policies traded not more than 30 days before expiration, we get the following chart. We can see that options with high premium provide no positive return, leading us to believe that the options with high premium are already in the money and the premium is fairly consistent with intrinsic value.

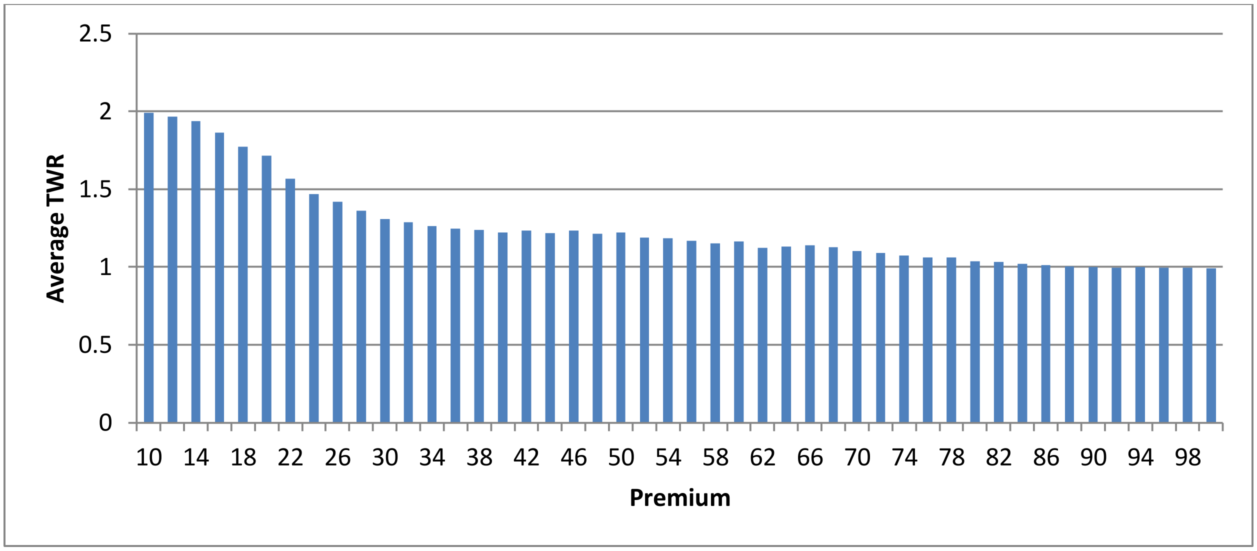


Figure 23 Average final TWR by premium for strategies trading at most 30 days before expiration

In the following chart, we use only policies traded at least 50 days before expiration. We can see the exaggeration of the hump around 75 points premium. This suggests that options with high premium can provide a reasonable return if purchased long before expiration.

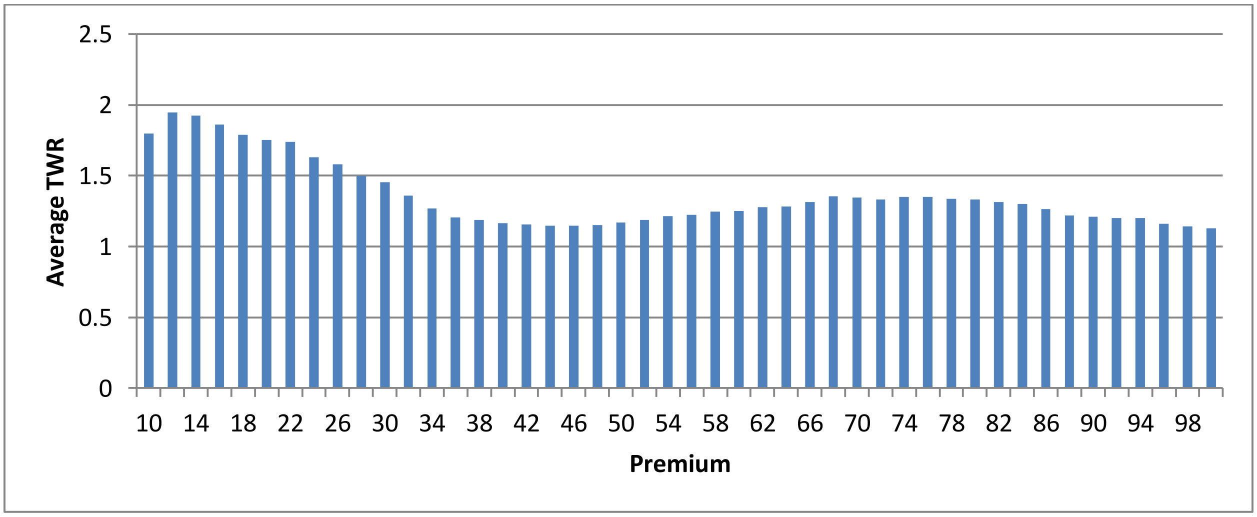


Figure 24 Average final TWR by premium for strategies trading at least 50 days before expiration

## 10.2 Premium and Days Before Expiration

The following chart, using Dataset B shows the average final TWR by both days to expiration and premium. The premium parameter is on the axis to the right and the days to expiration parameter is on the left. In general, TWR is pretty low except for days 24-28 and low premium.

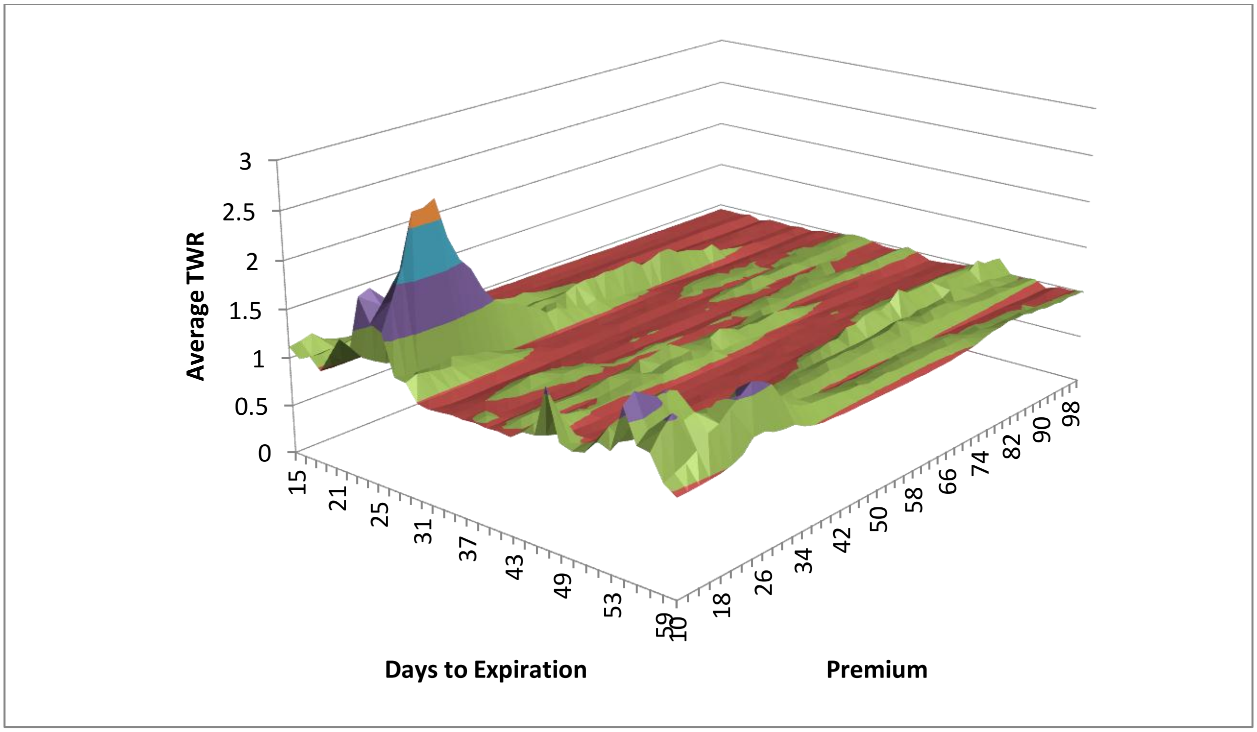


Figure 25 Surface of average final TWR by premium and days to expiration

## 10.3 Bear-Call/Bull-Put Increments

The difference between the bear-call strike and the call strike is parameterized by default as [5, 100] in increments of 5. The difference between the bull-put strike and put strike is parameterized in the same range. Additionally, we allow a strategy not to buy the bear-call or bull-put insurance. If the market is guaranteed to be bullish, a bull-put is unnecessary.

For this analysis we used Dataset B. First we show the TWR by each of the values for bear-call and bull-put. These are the differences in strike price between the outside and inside option.

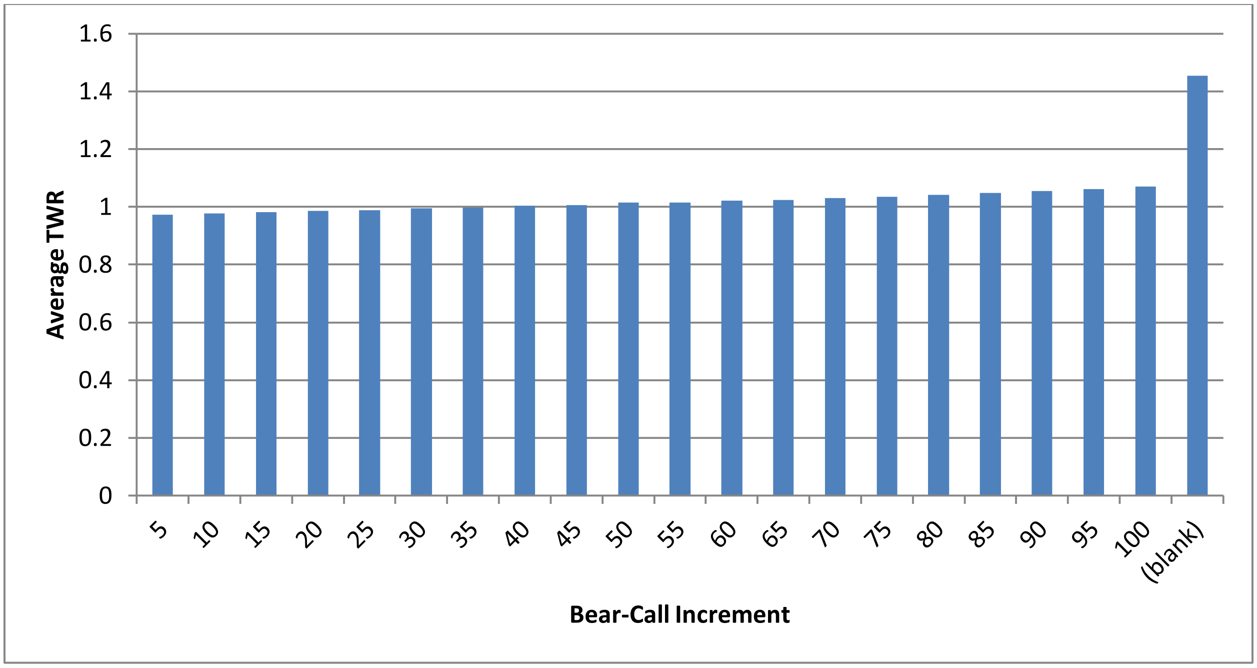


Figure 26 Average final TWR by bear-call strike price difference

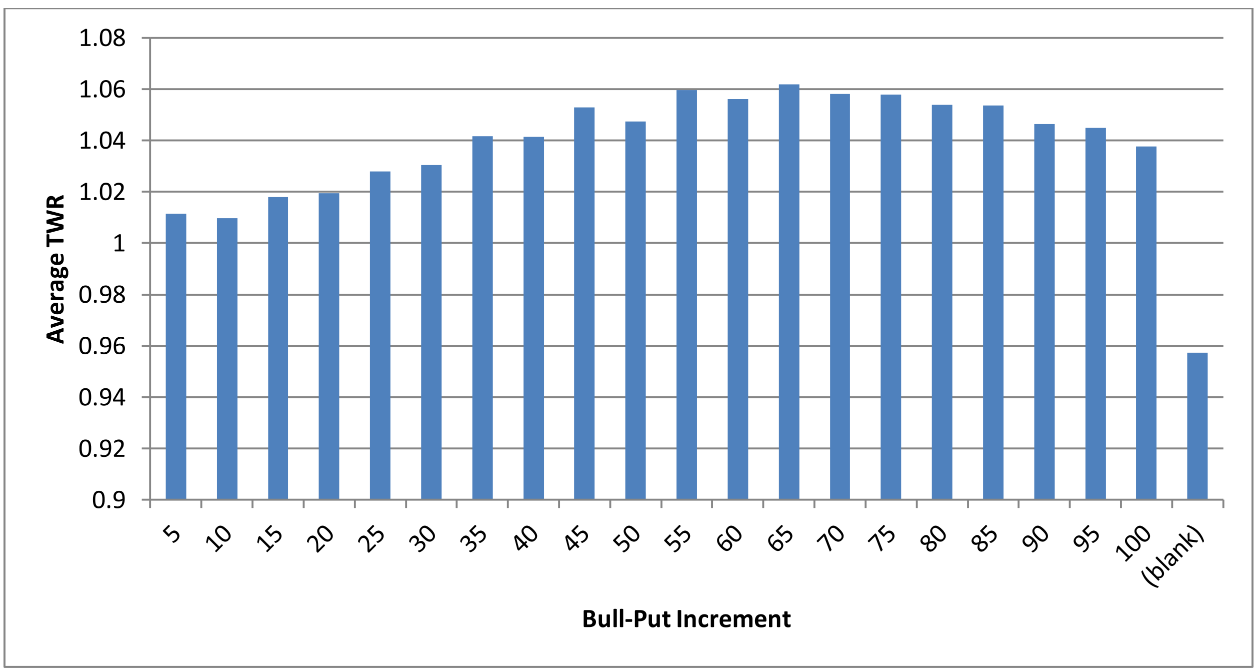


Figure 27 Average final TWR by bull-put strike price difference

These two charts indicate something particularly interesting about the market. The first chart shows that a bear-call is usually unnecessary. The best policies are those without any form of insurance on the high side. Bull-puts however, are necessary and the policies with the best return are those that purchased a bull-put between 40 and 100 points below the short put option. This indicates a bear market. Sudden steep drops in the market would make policies without a bull-put unprofitable. But slow gradual inclines in the market would not require a bear-call.

Since this analysis is over the 2007-2009 timeframe, it includes the crash of 2009 and the very volatile market for those few months. This analysis fits with the market during that time.

The following chart shows that no particular combination of bear-call and bull-put increments are especially profitable, except those with no bear-call (the lower axis). The top, or inner, axis shows the bull-put difference.

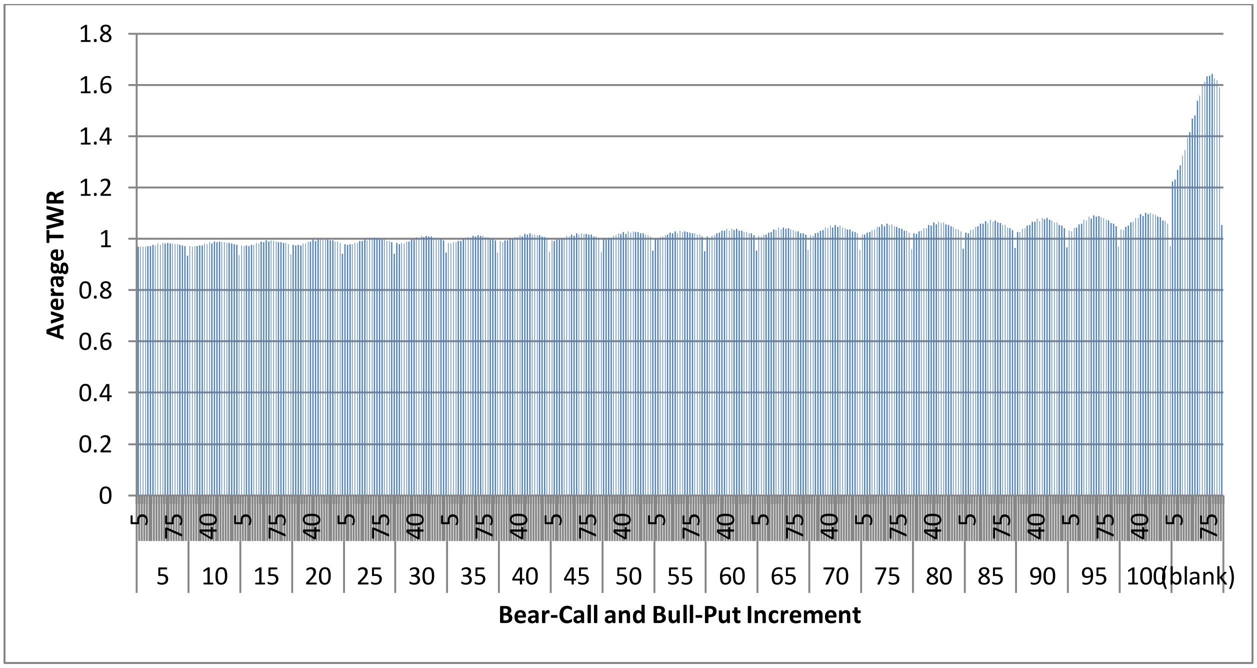


Figure 28 Average final TWR by bear-call and bull-put strike price differences

## 10.4 Impact of Slippage

For the above analyses, some were generated using Dataset A and others used Dataset B. When looking at these two datasets, the only difference is that Dataset B uses slippage while the other does not. For the above analyses, this change in parameter does not affect the conclusions we draw about optimal parameter values. This next chart shows why.

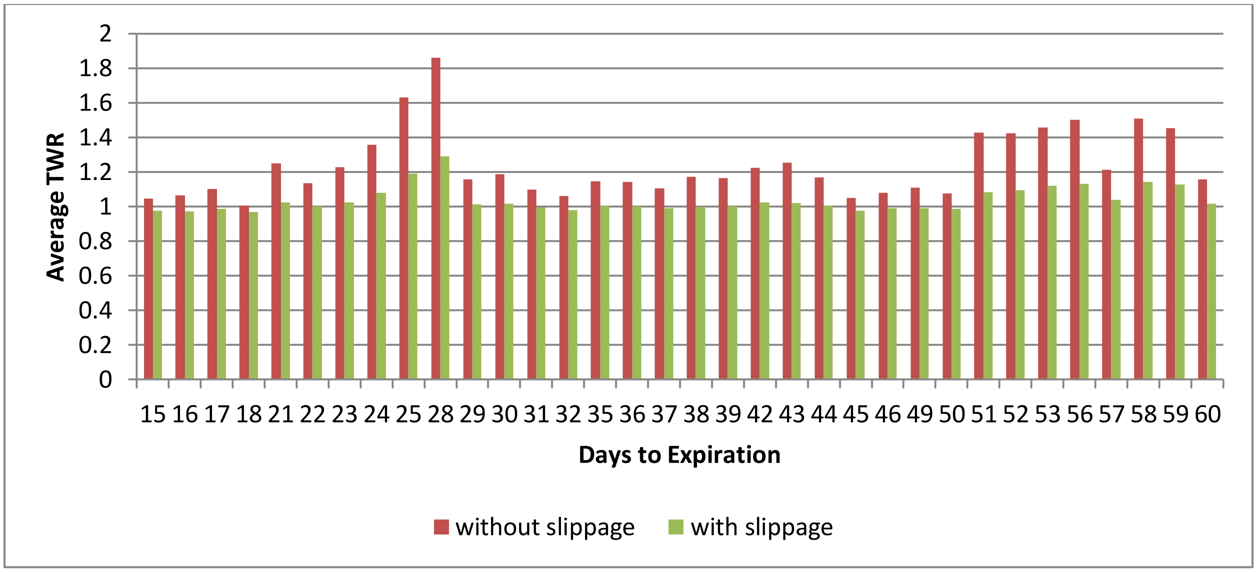


Figure 29 Average final TWR by days to expiration with and without slippage

When all other parameters are held constant the only contribution of slippage is to lower the final TWR proportionally for all policies. It does not change the ordering of the optimal strategies. As we can see in the chart above, days 25 and 28 are still the best days to trade, but the final TWR is lower when using slippage than it is without slippage. We could show similar charts for all other parameters but this motivates the conclusion that with or without slippage, the conclusions hold. In some cases we use the dataset without slippage to show greater variation in the final TWR.

## 10.5 Sensitivity Analysis

Using Dataset B, the following are the top 15 optimal policies. A quick scan shows that the days to expiration parameter is a constant 28, and the premium parameter is in the range of 16 to 24. None of the optimal policies use a bear-call. They all have a bull-put between -35 and -65 off the short put. This indicates that the strategy is quite stable in terms of the parameters we use.

| Days | Premium | Bear-Call Increment | Bull-Put Increment | Final TWR |
|---|---|---|---|---|
| 28 | 20 | | -55 | 9.05 |
| 28 | 20 | | -50 | 8.98 |
| 28 | 20 | | -45 | 8.68 |
| 28 | 20 | | -60 | 8.33 |
| 28 | 18 | | -45 | 8.28 |
| 28 | 20 | | -35 | 8.26 |
| 28 | 20 | | -40 | 8.18 |
| 28 | 18 | | -40 | 7.99 |
| 28 | 20 | | -65 | 7.99 |
| 28 | 22 | | -55 | 7.98 |
| 28 | 16 | | -40 | 7.90 |

| | | | | |
|---|---|---|---|---|
| 28 | 22 | | -45 | 7.84 |
| 28 | 18 | | -50 | 7.83 |
| 28 | 22 | | -60 | 7.79 |
| 28 | 24 | | -55 | 7.66 |

Table 3 Top 15 strategies showing the optimal parameters for trading over 2007-2009

Using an iron condor strategy 28 days before expiration, with a premium in the range of 16 to 24 and a bull-put about 35 to 65 points under the put, the returns will be pretty much consistent over the time period our team analyzed, 2007-2009.

We took exactly these parameters and ran those policies (approximately 30 of them) on each year, 2004 through 2009 individually. We then plotted the average TWR of these policies for each year to see how well they performed. The chart below shows how the policies which were optimal for 2007-2009 do not perform so well for other years. We also found the optimal set of policies for 2004 and ran these over each year 2005 through 2009. The performance of these policies again varied quite a bit, showing that policies that are optimal in one year do not necessarily perform best in other years.

The optimal set of policies for 2007-2009 had the following parameters: 28 days to expiration, premium of 16 to 22 in increments of 2, no bear call, bull-put spread of 35 to 65 in increments of 5 and no maximum volatility. The optimal set of policies for 2004 had the following parameters: 59 days to expiration, premium of 28 to 34 in increments of 2, no bull-put, bear-call spread of 50 to 100 in increments of 10 (including also no bear-call) and no maximum volatility.

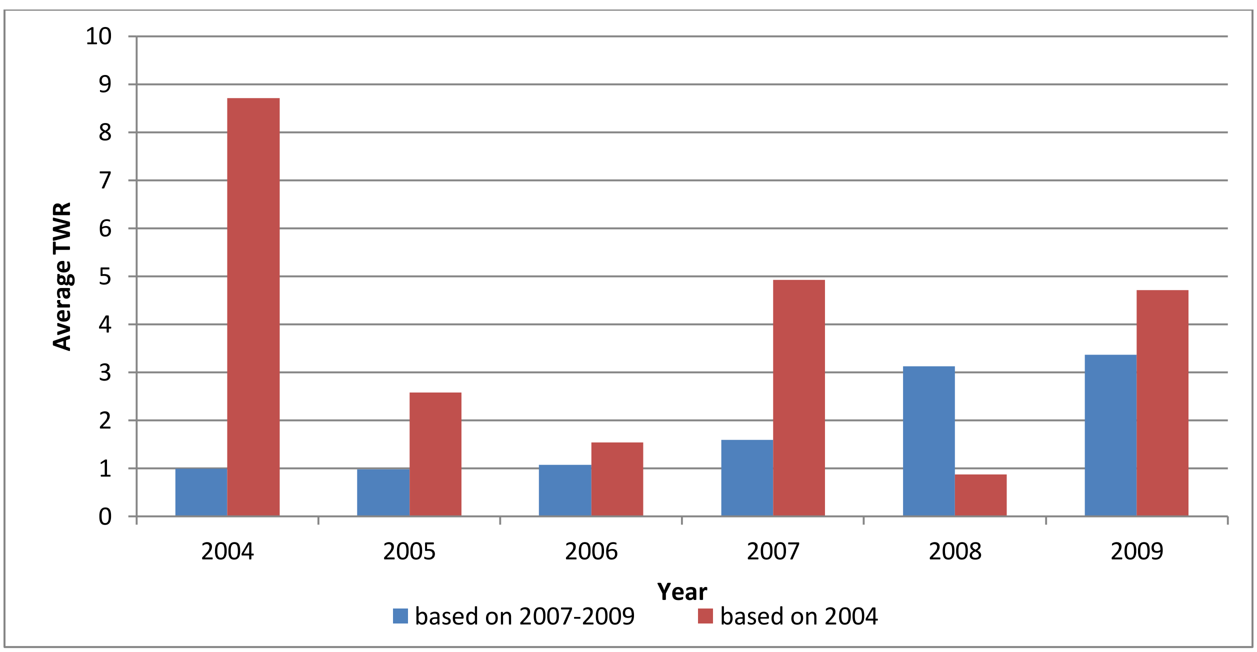


Figure 30 Year-over-year performance of strategies which are optimal for only one time period

# 11 Appendix – Software Functionality

As described in 3.1, our first task was to extend the existing trading simulation software. We will briefly show the graphical user interface we developed.

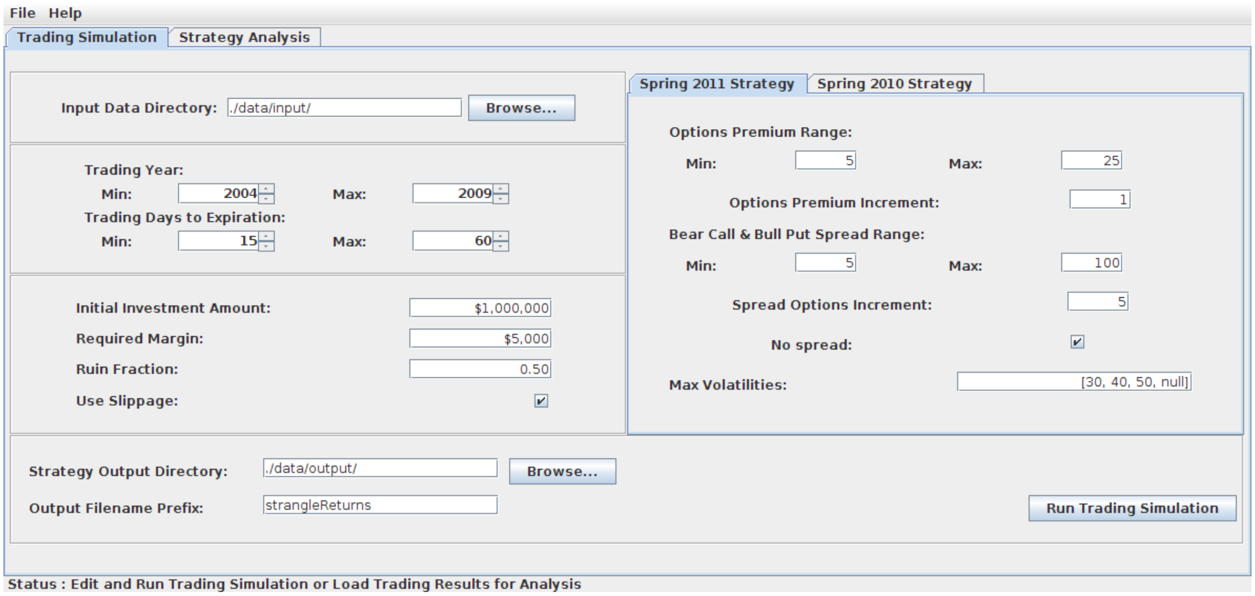


Figure 31 Main window for the simulation GUI

This screenshot of the software shows how the simulation is configured and run. There are two basic functions for this program, represented as tabs – Trading Simulation and Strategy Analysis. The simulation tab allows the user to configure a new simulation and execute it. The analysis tab shows the results of all strategies and allows the user to view the performance of each one individually, or compare multiple policies.

### 11.1.1 Trading Simulation Tab

The Trading Simulation Tab configures all possible parameters for the trading simulation. On the left are general parameters for all types of strategies. The user enters the date range. The years of 2004-2009 have the most useful data. The user also configures how many days before expiration to trade – this is a range and each day within this range is added as a possible trading day. These are calendar days, but of course Saturday, Sunday and trading holidays are ignored. So not all 60-15=45 (default) trading days are valid.

Next the user can configure the Initial Investment Amount. The Required Margin is a parameter that comes from the index we're trading. The E-mini requires a $5,000 margin. The Ruin Fraction determines the fraction of initial investment the user is willing to lose before considering himself ruined. And slippage is a parameter we added that enables or disables slippage in trading. Without slippage, there is no transaction cost, no matter how large the transaction.

The previous software workflow required the user to execute a simulation and output the results to a file, then load the file using the analysis software to view and analyze. We also allow the user to save the results to a directory. The program can then load the previous run without rerunning the simulation.

On the right, the user can choose between two different strategies. More can be added using the existing Java framework. The Spring 2010 Strategy allows the user to enter the parameters the previous team used, such as strike price differences for put and call, stop-loss values and maximum volatility.

Our Spring 2011 Strategy shows that the possible parameters are premium range, spread range and maximum volatility. We also allow the user to attempt a strategy without any spread option (bear or bull).

### 11.1.2 Strategy Analysis Tab

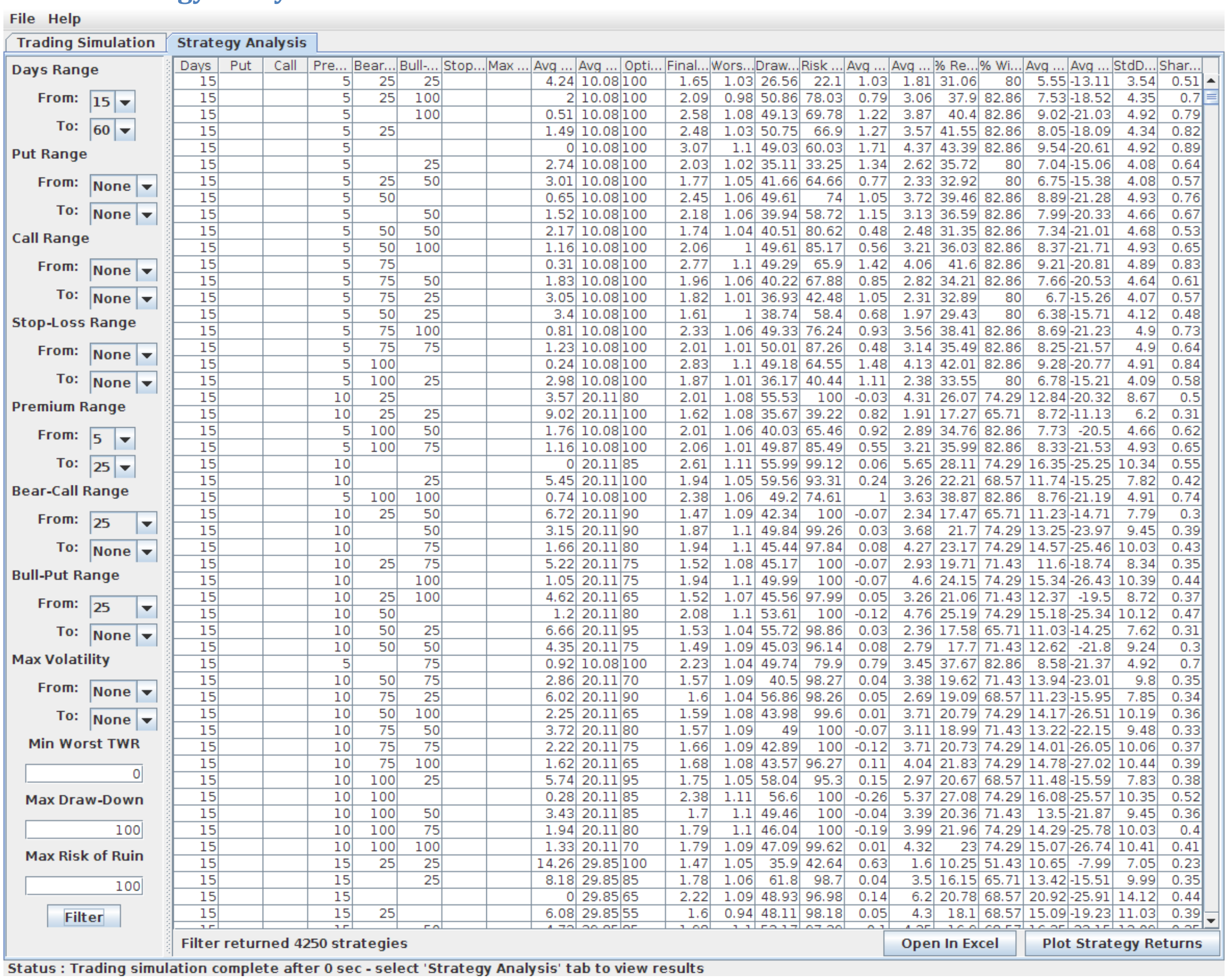


Figure 32 Strategy Analysis Tab showing simulation results

The analysis tab contains mainly a single table where the user can view the results of all the policies the simulation ran. The panel on the left allows the user to filter the results by each of the possible parameters. There is an Open in Excel button that exports the results, with additional columns and data, to Excel to allow further analysis and data sharing.

If the user selects a set of rows and presses the Plot Strategy Returns button, the following window comes up, showing the returns over time. (Much of this functionality was implemented by the previous team in spring of 2010 (Chen, et al. 2010).) Not all trades may be shown in this graph. In this example, 28 days before the third Friday in March, 2008, the CME was closed for Good Friday. Since no trade data existed for that day, the simulation did not execute a trade.

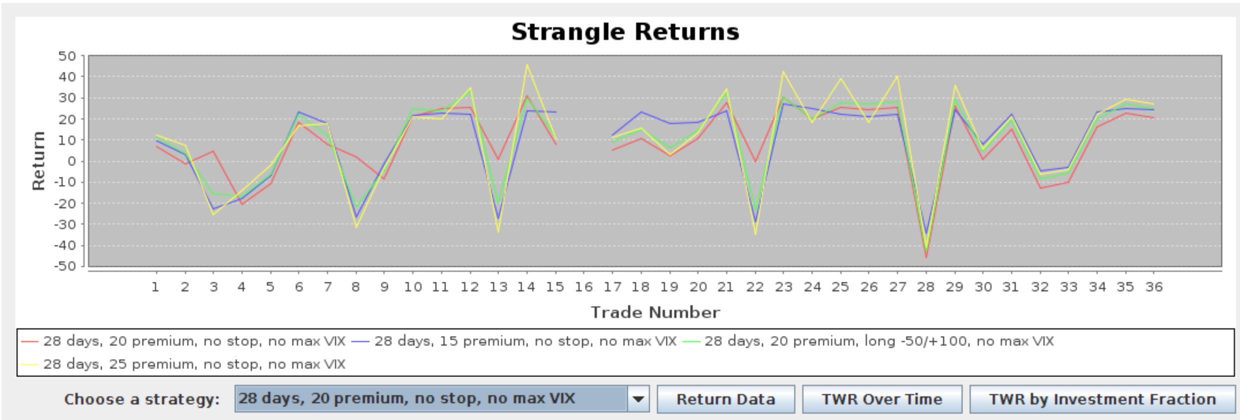


**Figure 33 Plot of returns over time using the analysis software**

The TWR over Time is especially interesting, showing the return for any of the investment fractions 5% through 100% and also Kelly's fractional allocation. This shows a single strategy and its returns over 2007-2009 with $10 million initial investment and slippage. An investment of $10 million is a large portion of the market and slippage causes the 100% investment fraction to produce suboptimal returns. You can see the light purple dip down lower than other during trade 33. This chart also shows the Kelly fraction as it changes over time.

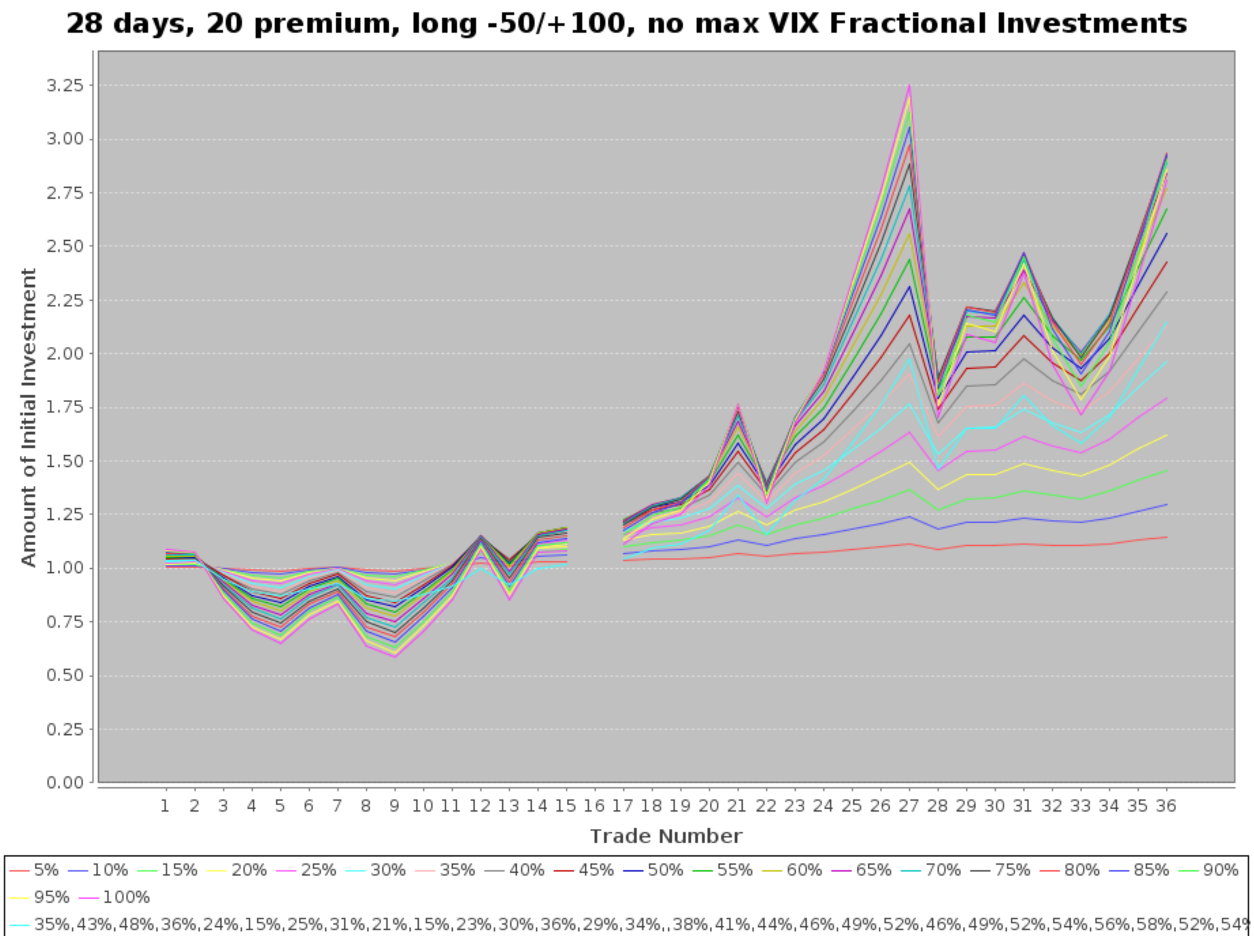


Figure 34 Plot of returns using each possible investment fraction using the analysis software